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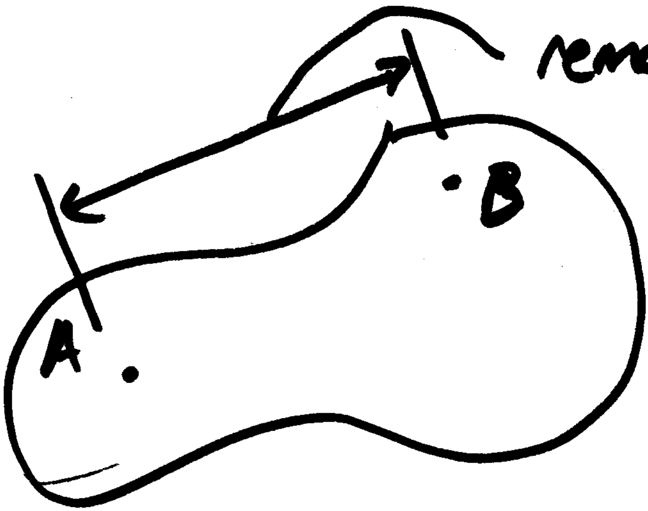
Chapter 16 Planar Kinematics of a Rigid Body:

Rigid Body: Body of finite extent, Particles in body remain in fixed relative position

remains fixed.



University of Idaho Why do we care??

Because Newton's 2nd Law is

$$\vec{F} = m\vec{a}_G \qquad \vec{a}_G = \text{Acceleration of the center of gravity.}$$

In many situations, our given information about acceleration of a rigid body is for a point on the body that is not the center of mass. So, we have to relate the given acceleration to $\vec{a}_G$.

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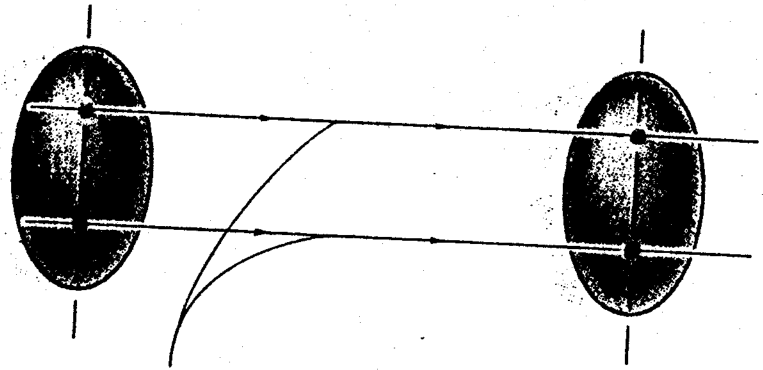
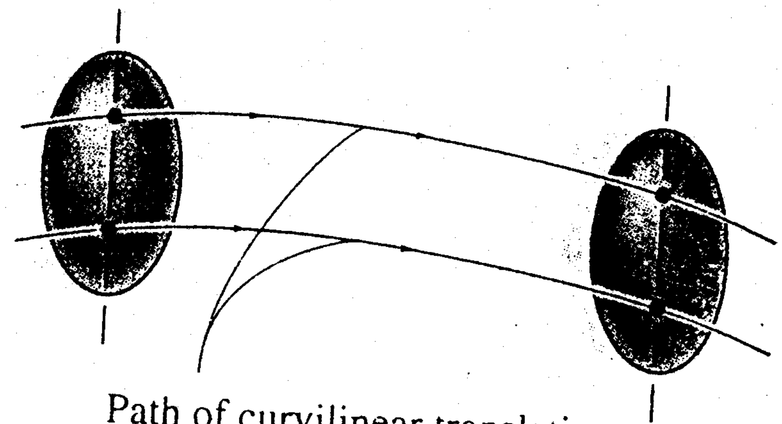


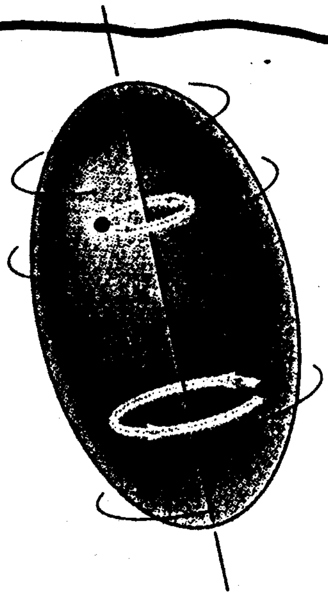
Path of rectilinear translation

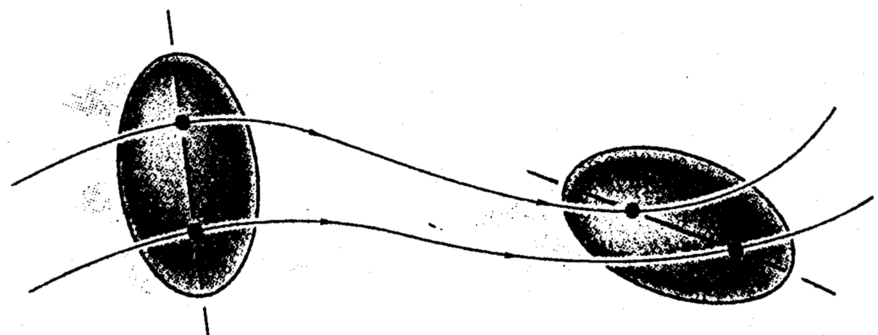
(a)



Path of curvilinear translation

(b)





General Plane Motion

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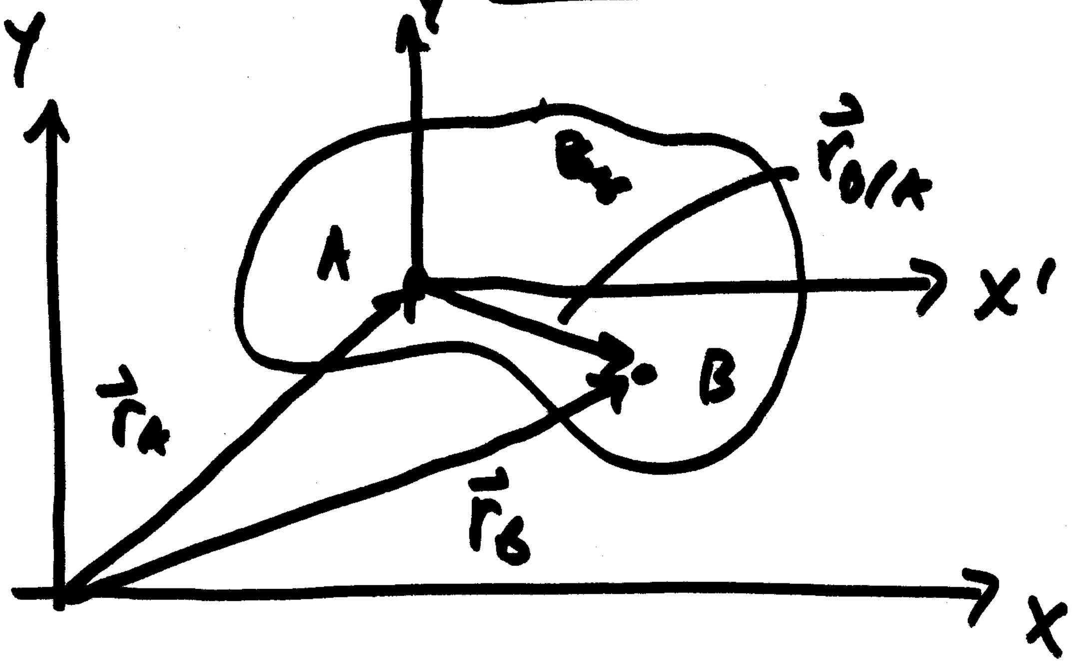
Translation



xy fixed

x'y' translates with A, but <u>does not</u> rotate

$\vec{i}, \vec{j}$ attached to xy

$\vec{i}', \vec{j}'$ attached to x'y'

Choose x'y' ∥ xy Then $\vec{i} = \vec{i}', \vec{j} = \vec{j}'$

$$\vec{r}_A = x_A \vec{i} + y_A \vec{j}$$

$$\vec{r}_B = x_B \vec{i} + y_B \vec{j}$$

$$\vec{r}_{B/A} = x'_{B/A} \vec{i}' + y'_{B/A} \vec{j}'$$

$$= x'_{B/A} \vec{i} + y'_{B/A} \vec{j}$$

University of Idaho Form the vector triangle

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

Wish to calculate velocity, take d/dt of the vector triangle: $\left[d/dt\,(\vec{i},\vec{j}) = 0 \right]$

$$\frac{d\vec{r}_B}{dt} = \dot{x}_B\vec{i} + \dot{y}_B\vec{j} = \vec{v}_B = \vec{v}_A + \cancelto{0}{\vec{v}_{B/A}}$$

Because its a rigid body

And similarly, apply d/dt again, and ~~recog~~ recognizing that $\vec{a}_{B/A} = 0$:

$$\left.\vec{a}_B = \vec{a}_A\right\} \therefore$$ If we know velocity and acc'n at one point on a <u>translating</u> body, we know them at any other point.

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University of Idaho **Fixed Axis Rotation (Section 16.3)**

~~Front~~ Translation (~~Horiz~~ Section 16.2):

Now we consider the case where the body is rotating, but not translating

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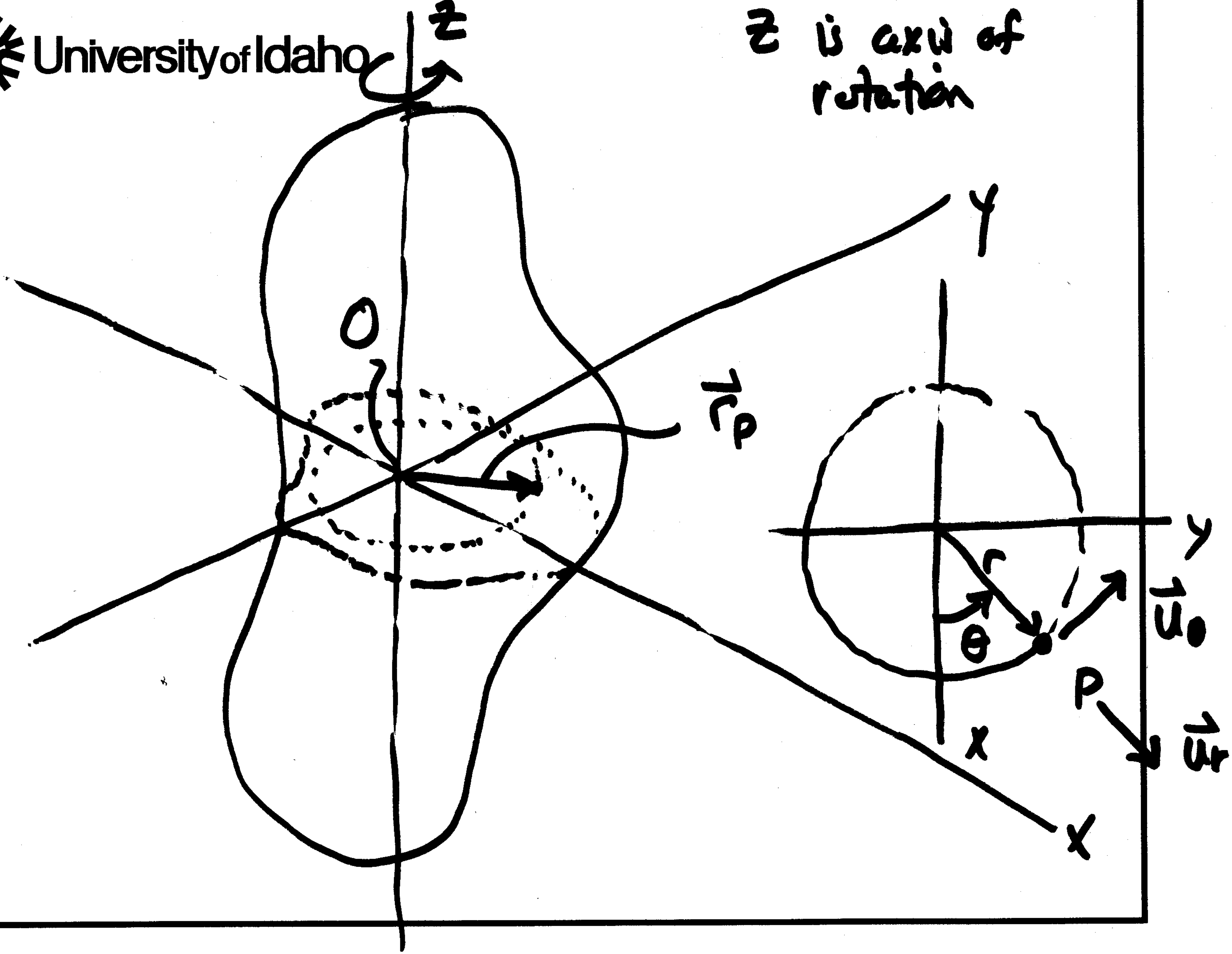
z is axis of rotation



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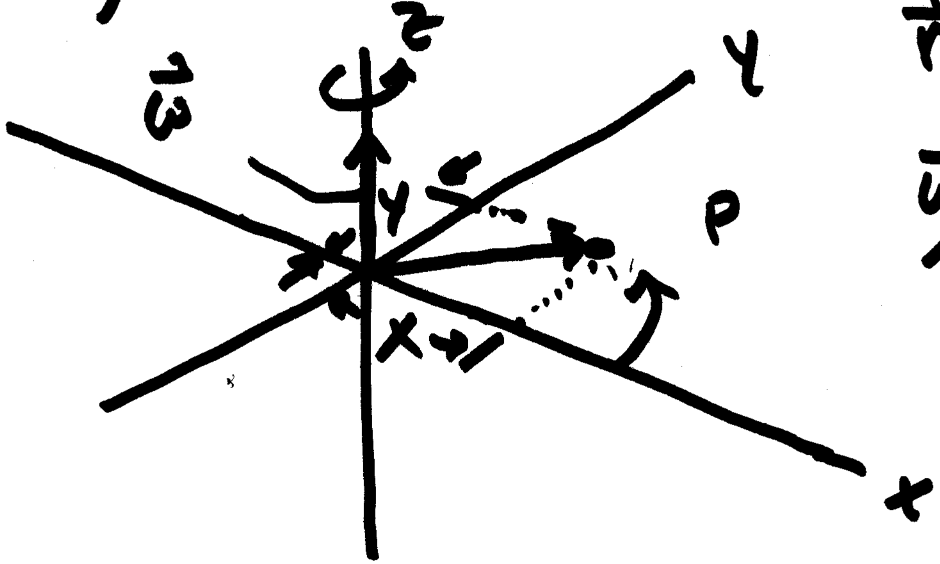
And, the velocity of pt P is

$$\vec{v} = \cancel{\dot{r}\vec{u}_r}^{\,0} + r\dot{\theta}\vec{u}_\theta = r\dot{\theta}\vec{u}_\theta \quad : \quad \frac{d\theta}{dt} = \dot{\theta} = \omega = \text{angular speed } \frac{\text{rad}}{\text{sec}}$$

Rigid body

$$= r\omega\vec{u}_\theta$$

Another way to look at this is:



$$\vec{r}_p = x\vec{i} + y\vec{j}$$

$$\vec{\omega} = \dot{\theta}\vec{k} = \omega\vec{k}$$

↑ angular velocity vector.

University of Idaho Then, the velocity is also given by

$$\vec{V} = \vec{\omega} \times \dot{\vec{r}}_p \qquad |\vec{V}| = |\vec{\omega} \times \vec{r}_p|$$

$$= |\vec{\omega}| \cdot |\vec{r}_p| \cdot \sin 90^\circ$$

Lets carry out the cross-product

$$\vec{\omega} = \omega \vec{k}$$
$$\vec{r}_p = x\vec{i} + y\vec{j}$$

$$\vec{\omega} \times \vec{r}_p = \begin{array}{cc|ccc|cc} j & k & i & j & k & i & j \\ 0 & \omega & 0 & 0 & \omega & 0 & 0 \\ y & 0 & x & y & 0 & x & y \end{array}$$

$$= (0 - y\omega)\vec{i} + (\omega x - 0)\vec{j} + (0 - 0)\vec{k}$$

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$$\vec{V} = -y\omega\vec{i} + \omega x\vec{j}$$

$$|\vec{V}| = \sqrt{\omega^2 y^2 + \omega^2 x^2} = \omega\sqrt{x^2+y^2} = r\omega \ldots\ldots$$

For acceleration; in cylindrical coordinates, we get

$$\vec{a} = (\cancel{\ddot{r}}^{\,0} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\cancel{\dot{r}}^{\,0}\dot{\theta})\vec{u}_\theta$$

Rigid body

$$\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \underset{\substack{\uparrow \\ \text{angular accln} \\ \text{rad/s}^2}}{\alpha}$$

$$\vec{a} = -r\dot{\theta}^2\vec{u}_r + r\ddot{\theta}\,\vec{u}_\theta = \underset{\substack{\uparrow \\ a_n}}{-r\omega^2\vec{u}_r} + \underset{\substack{\uparrow \\ a_t}}{r\alpha\,\vec{u}_\theta}$$

University of Idaho Again, given rectangular coordinate;

From $\vec{V} = \vec{\omega} \times \vec{r}_p$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r}_p + \vec{\omega} \times \frac{d\vec{r}_p}{dt}$$

; $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$ = angular acceleration vector

$$= \vec{\alpha} \times \vec{r}_p + \vec{\omega} \times \vec{V}$$

$$\vec{a} = \underbrace{\alpha \times \vec{r}_p}_{\text{tangential acceleration}} + \underbrace{\vec{\omega} \times \vec{\omega} \times \vec{r}_p}_{\text{normal acceleration}}$$