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University of Idaho Exam I Ptcle Kinematics :

Kinetics ; Wed Sept. 29, Session 16

Open Book, Open Notes, Open Technology

~~Rectint~~ Rectilinear Motion:

displacement s, $v = \frac{ds}{dt}$, $a = \frac{d^2s}{dt^2} = \frac{dv}{dt}$

$a(s)ds = v dv$

Integral forms as well

Constant acceleration $a = a_c$

$v = v_0 + a_c t$, $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$, $v^2 = v_0^2 + 2a_c(s - s_0)$

University of Idaho Erratic Motion - Integrations over segments of time or distance (Rectilinear)

$v = \frac{ds}{dt}$ ⇒ <u>given $s(t)$</u> ; create $v(t)$

$a = \frac{dv}{dt}$ ⇒ <u>given $v(t)$</u> ; create $a(t)$

<u>Given $a(t)$</u> ⇒ $v - v_0 = \int_{t_0}^{t} a(t)\,dt$, creating $v(t)$

<u>Given $v(t)$</u> ⇒ $s - s_0 = \int_{t_0}^{t} v(t)\,dt$, create $s(t)$

<u>Given $a(s)$</u> ⇒ $\int_{s_0}^{s} a(s)\,ds = \frac{1}{2}v^2 - \frac{1}{2}v_0^2$, creating $v(s)$

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Specification of Curvilinear Motion in Rectangular Coordinates

$$\vec{r} = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

$$\vec{v} = \dot{x}\,\vec{i} + \dot{y}\,\vec{j} + \dot{z}\,\vec{k}$$

$$\vec{a} = \ddot{x}\,\vec{i} + \ddot{y}\,\vec{j} + \ddot{z}\,\vec{k}$$

$$|\vec{v}| = v = \text{speed} = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

$$|\vec{a}| = a = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2} = \text{magnitude of acceleration}$$

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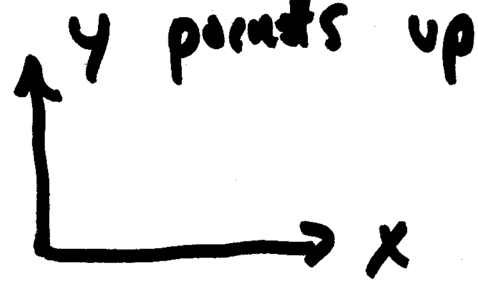
Projectile Motion



Horizontal

$v_x = (v_0)_x$

$x = x_0 + (v_0)_x t$

Vertical ↑

$v_y = (v_0)_y - g t$

$y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$

$v_y^2 = (v_0)_y^2 - 2g(y - y_0)$

$\vec{v}(t) = v_x \vec{i} + v_y \vec{j}$

$\vec{r}(t) = x \vec{i} + y \vec{j}$

Initial Velocity, Pos Vectors ⇒

$\vec{v}_0 = (v_0)_x \vec{i} + (v_0)_y \vec{j}$

$\vec{r}_0 = x_0 \vec{i} + y_0 \vec{j}$

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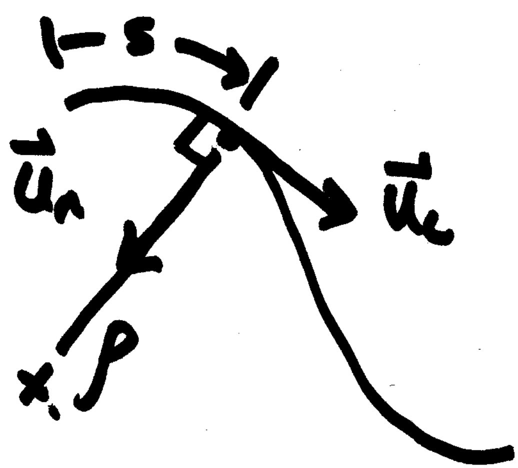
Curvilinear Motion, Normal & Tangential Components



$$\vec{v} = \dot{s}\,\vec{u}_t = v\,\vec{u}_t$$

$$\vec{a} = \dot{v}\,\vec{u}_t + \frac{v^2}{\rho}\,\vec{u}_n$$

For $\dot{v} = a_c = \text{constant}$

$$v = v_0 + a_c t$$

$$s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$\rho = \frac{\left[1 + dy/dx\right]^{3/2}}{\left|d^2y/dx^2\right|}$$ evaluated at ptcle position.

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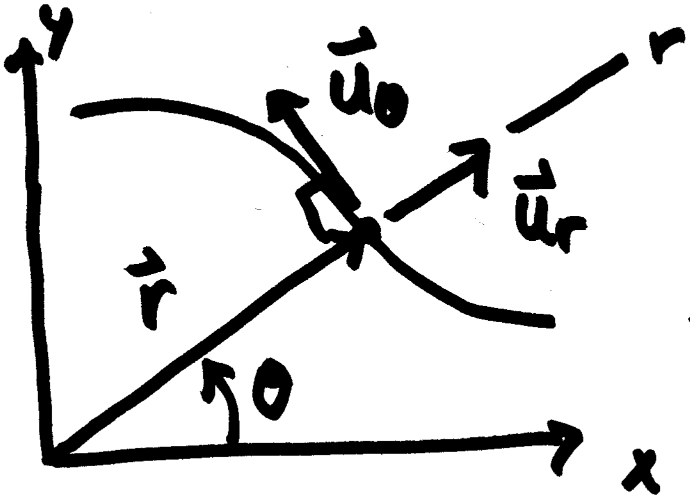
Curvilinear Motion - Cylindrical Coordinates



$$\vec{r} = r\vec{u}_r$$

$$\vec{v} = \dot{r}\vec{u}_r + r\theta\,\vec{u}_\theta$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta$$

Chain Rule for Differentiation, e.g, for $\theta = \theta(t)$

no dot

$\cancel{\frac{d}{dt}[\sin}$ $\quad \frac{d}{dt}\left\{\sin[\theta(t)]\right\} = \cos[\dot{\theta}(t)] \cdot \dot{\theta}(t)$

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Ptcle Kinetics

$\vec{F} = m\vec{a}$

↑

net resultant of applied external forces

Rect Coord

$F_x = ma_x = m\ddot{x}$

$F_y = ma_y = m\ddot{y}$

$F_z = ma_z = m\ddot{z}$

Normal Tangent Coords

$F_n = ma_n = m\frac{v^2}{\rho}$

$F_t = ma_t = m\dot{v}$

Cylindacal Coords

$F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$

$F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

University of Idaho Problem Solving in Ptcle Kinetics

① Draw a FBD for each ptcle. Assign a coord system to each FBD

② Assign external applied forces to each body. Originate from contact or gravity

③ Write Newton's Laws. for each body.

④ Count knowns, unknowns & eqns.

⑤ Supplement Newton's Laws if nec with kinematic constraints

⑥ Solve for the unknowns.

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Mass in English Units $m = \frac{W \; lb_f / lb_m}{g = 32 \; ft/s^2}$

What's Not on The Exam

⇒ Pulley problems

⇒ No 3D problems

⇒ No too long problems

⇒ No relative motion problems.