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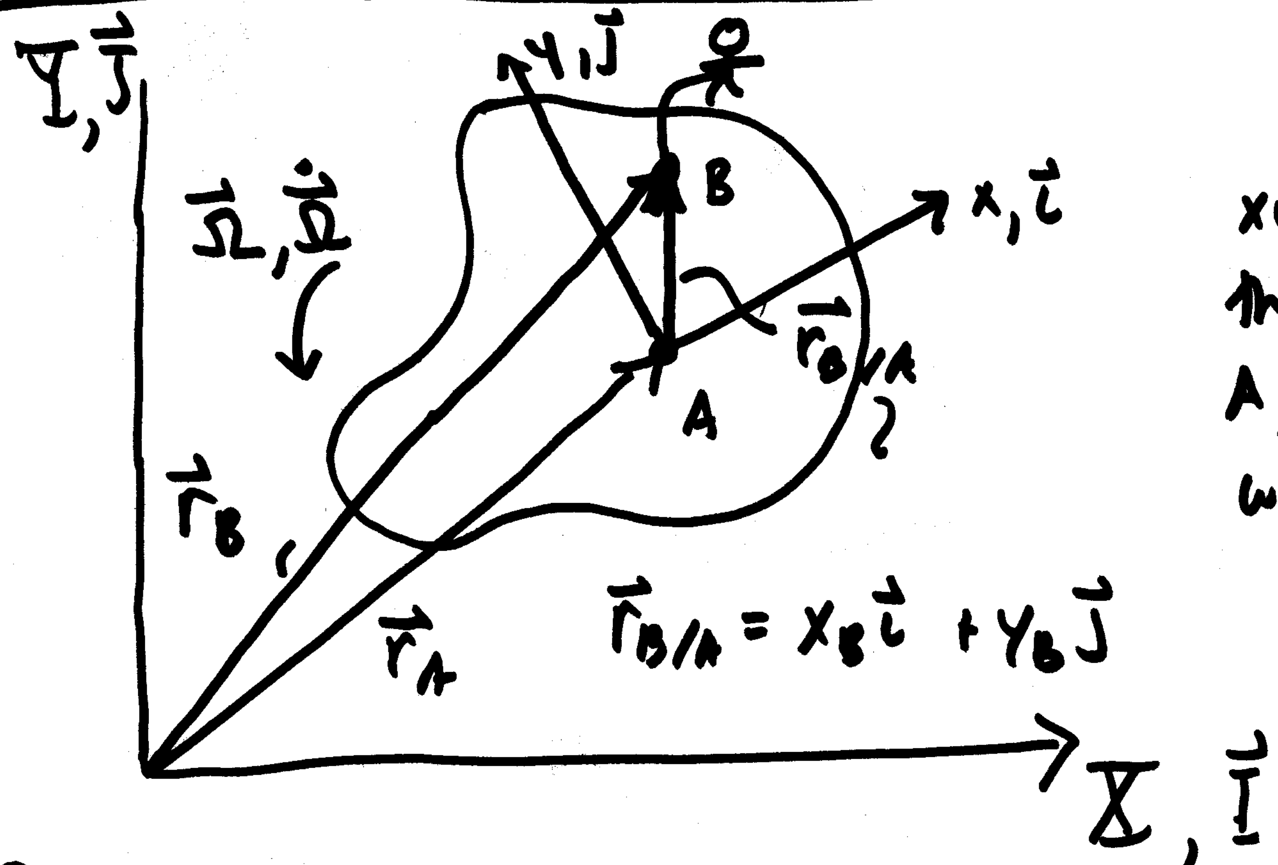
Section 16.8 Relative Motion

Analysis Using Rotating Axes



$\vec{r}_{B/A} = x_B\vec{i} + y_B\vec{j}$

XY is a fixed frame

xy is fixed to the rigid body at A, and rotates with the rigid body.

Observer presently at B, moving relative to B.

$\vec{r}_A$ locates point A in the fixed XY frame

$\vec{r}_{B/A}$ locate point B relative to point A in the xy frame. Locates at the present instant, in the rotating frame xy.

$\vec{r}_B$: the ~~vector~~ present position of in the fixed frame.

Objective: $\frac{d\vec{r}_B}{dt} = \vec{v}_B$ = velocity of in the fixed frame.

$\frac{d\vec{v}_B}{dt} = \vec{a}_B$ = acceln of in the fixed frame

University of Idaho Want to compute $\frac{d\vec{r}_B}{dt}$:

$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

$\cancel{\frac{d\vec{r}_B}{dt} = \vec{V}_B} =$

$$\frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{r}_{B/A}}{dt}$$

$$\vec{V}_B = \vec{V}_A + \frac{d}{dt}\left(x_B\vec{i} + y_B\vec{j}\right)$$

$$\vec{V}_B = \vec{V}_A + \frac{dx_B}{dt}\vec{i} + x_B\frac{d\vec{i}}{dt} + \frac{dy_B}{dt}\vec{j} + y_B\frac{d\vec{j}}{dt}$$

$$\vec{V}_B = \vec{V}_A + \underbrace{\frac{dx_B}{dt}\vec{i} + \frac{dy_B}{dt}\vec{j}}_{(\vec{V}_{B/A})_{xyz}} + \underbrace{x_B\frac{d\vec{i}}{dt} + y_B\frac{d\vec{j}}{dt}}_{\text{Motion of point B}}$$

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$$\vec{V}_B = \vec{V}_A + \underbrace{(\vec{V}_{B/A})_{xyz}}_{\text{Motion of B relative to rotating frame}} + \underbrace{\vec{\Omega} \times (\vec{r}_{B/A})_{xyz}}_{\text{Motion of point B relative to fixed frame}}$$

$\vec{V}_B$ = velocity of B relative to the fixed frame.

$\vec{V}_A$ = velocity of A relative to the fixed frame.

$\vec{\Omega}$ = Angular velocity of the rigid body relative to the fixed frame.

$\vec{r}_{B/A}$ = Position of B rel to A measured in the rotating frame.

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The acceleration is found by differentiating the velocity

$$\frac{d\vec{v}_B}{dt} = \vec{a}_B = \frac{d\vec{v}_A}{dt} + \frac{d}{dt}(\vec{v}_{B/A})_{xyz} + \frac{d}{dt}\left[\vec{\Omega} \times (\vec{r}_{B/A})_{xyz}\right]$$

$$\vec{a}_B = \vec{a}_A + \left[(\vec{a}_{B/A})_{xyz} + \vec{\Omega} \times (\vec{v}_{B/A})_{xyz}\right]$$

$$+ \frac{d\vec{\Omega}}{dt} \times (\vec{r}_{B/A})_{xyz} + \vec{\Omega} \times \left[\frac{d}{dt}(\vec{r}_{B/A})_{xyz}\right]$$

$$= \vec{a}_A + \left[(\vec{a}_{B/A})_{xyz} + \vec{\Omega} \times (\vec{v}_{B/A})_{xyz}\right] + \dot{\vec{\Omega}} \times (\vec{r}_{B/A})_{xyz}$$

$$+ \vec{\Omega} \times \left[(\vec{v}_{B/A})_{xyz} + \vec{\Omega} \times (\vec{r}_{B/A})_{xyz}\right]$$

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$$\vec{a}_B = \vec{a}_A + \underbrace{\dot{\vec{\Omega}} \times (\vec{r}_{B/A})_{xyz} + \vec{\Omega} \times \vec{\Omega} \times (\vec{r}_{B/A})_{xyz}}_{\text{accln of point B relative to fixed frame}}$$

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$$+ 2\vec{\Omega} \times (\vec{v}_{B/A})_{xyz} + \underbrace{(\vec{a}_{B/A})_{xyz}}_{\text{accln of relative to rotating frame}}$$

$\vec{a}_B$ = Accln of relative to the fixed frame.

$\vec{a}_A$ = Accln of pt A relative to the fixed frame.

$\dot{\vec{\Omega}}$ = Angular acceleration of the rigid body, rotating frame

$(\vec{a}_{B/A})_{xy}$ = Accln of relative to the rotating frame.