University of Idaho Sections 12.1 - 12.2

Introduction, Rectilinear Kinematics: Continuous Motion

Rectilinear Kinematics of a Particle

Particle: ~~Eta~~ Idealization of a body whose mass is concentrated at at a point (center of mass)

One dimensional motion; measure position, velocity, and accln of a particle along a straight line.

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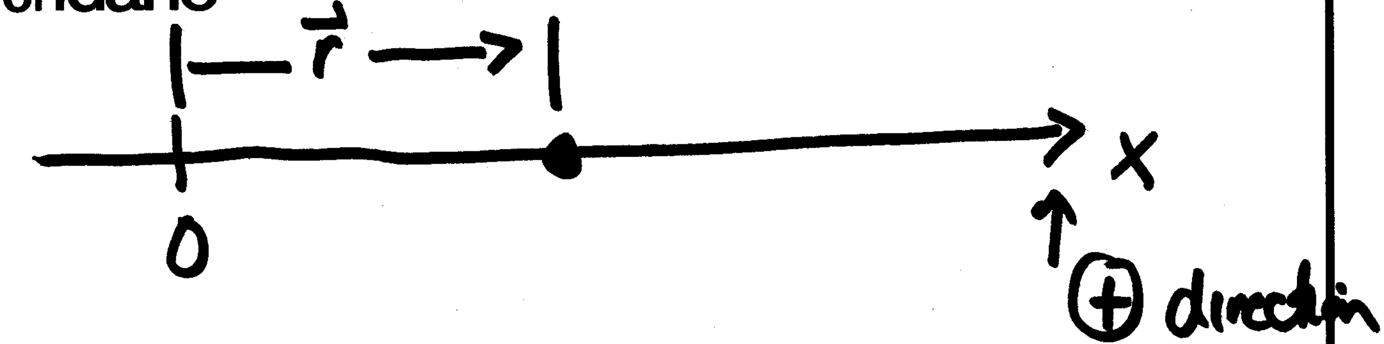
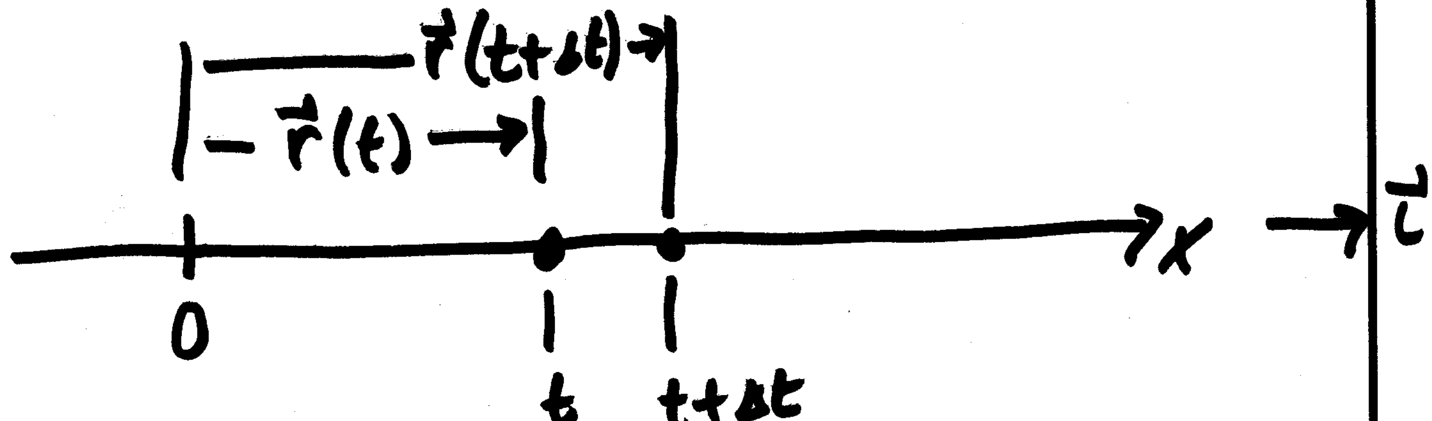
Position



Position vector ; $\vec{r} = s\,\vec{i}$, $\vec{i}$ = unit vector unit vector in $\oplus$ x direction

s = magnitude of the position vector = position

Velocity



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Velocity vector $\vec{v} = \lim_{\Delta t \to 0} \dfrac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$

$$= \frac{d\vec{r}}{dt}$$

Since $\vec{r} = s\vec{\iota}$; $\dfrac{d\vec{r}}{dt} = \dfrac{ds}{dt}\vec{\iota} \qquad \vec{\iota} \neq \vec{\iota}(t)$

$$= \dot{s}\,\vec{\iota} \qquad \text{"}\,\dot{}\,\text{"} = \frac{d}{dt}$$

The magnitude $|\vec{v}| =$ speed ; speed $= |\dot{s}|$

$$v = \frac{\text{lower}}{\text{case}} = |\vec{v}| = |\dot{s}|$$

Acceleration : $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}(\dot{s}\,\vec{\iota}) = \ddot{s}\,\vec{\iota}$; m/s^2

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$\ddot{s}$ can be (+), means velocity is increasing, or $\ddot{s}$ can be (−), which means velocity is decreasing

$\vec{v} = \frac{d\vec{r}}{dt}$; $\boxed{v = \frac{ds}{dt}}$ $\qquad dt = \frac{ds}{v}$

$\vec{a} = \frac{d\vec{v}}{dt}$: $\vec{a} = a\vec{\tau}$; $\boxed{a = \frac{dv}{dt}}$ $\Rightarrow$ $dt = \frac{dv}{a}$

$\Rightarrow \frac{ds}{v} = \frac{dv}{a} \Rightarrow \boxed{a\,ds = v\,dv}$

$$\int_{s_0}^{s} a(s)\,ds = \int_{v_0}^{v} v\,dv$$

 IC's ~~at~~ when ptcle is at position s_0, it has velocity v_0. The result of the integration will be the velocity v at <u>any given</u> s.

Consider the case of constant acceleration $a = a_c$

$$a_c = \frac{dv}{dt} \Rightarrow \int_{t_0}^{t} a_c\,dt = \int_{v_0}^{v} 1\,dv$$

IC's v_0 @ time t_0 ; $a_c(t - t_0) = v - v_0$

$$\boxed{v = a_c(t - t_0) + v_0} \qquad t_0 = 0$$

University of Idaho Apply again:

$$v = \frac{ds}{dt} \Rightarrow \cancel{a_c(t-t_0)} \quad t_0 = 0$$

$$v = a_c t + v_0 = \frac{ds}{dt} \; ; \quad (a_c t + v_0)\,dt = ds$$

$$\int_0^t (a_c t + v_0)\,dt = \int_{s_0}^{s} ds \Rightarrow a_c \frac{t^2}{2} + v_0 t = s - s_0$$

$$\boxed{s = a_c \frac{t^2}{2} + v_0 t + s_0}$$

s_0, v_0 are position, velocity at time $t = t_0 = 0$

University of Idaho **Third constant acceln tools:**

$$a\,ds = v\,dv \Rightarrow \int_{s_0}^{s} a_c\,ds = \int_{v_0}^{v} v\,dv$$

The particle has velocity v_0 at ~~posn~~ position s_0.

$$\boxed{a_c(s-s_0) = \frac{v^2}{2} - \frac{v_0^2}{2}}$$