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Example Problem 12-7

Given Position of a particle versus time is

$s = t^3 - 9t^2 + 15t$, ft during the time interval

$0 \leq t \leq 10$ s.

Find

- Maximum acceleration a
- Maximum velocity v

Tools/Approach $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$

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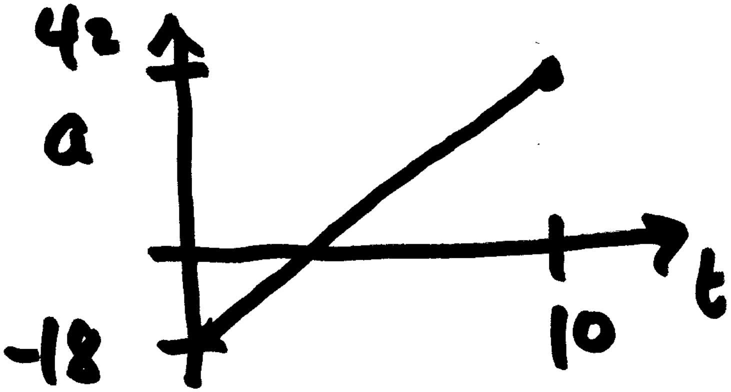
Compute vtt v vs. t, a vs t, determine max v and a by differentiation, plotting, or inspection.

$\boxed{\text{Solution}}$

$$v = \frac{ds}{dt} = 3t^2 - 18t + 15$$

$$a = \frac{dv}{dt} = 6t - 18$$



Maximum acceleration

$\boxed{a = 42 \text{ ft/s}^2}$

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We know $\frac{dv}{dt} = 0$ @ $t = 10\text{sec}$

$v(10) = 3 \cdot 100 - 18 \cdot 10 + 15 = 135 \text{ ft/sec}$

$$\boxed{V_{max} = 135 \text{ ft/sec}}$$

Judge Validity

- Plot a, v and verify that maxima occur at the proper times

University of Idaho **Example Problem 12-18**

Given: • ~~As~~ A car starts from rest ($t = 0$ sec)

• As it moves $a = 3s^{-1/3}$ m/s², s in m

Find: • Determine the acceleration of the car when $t = 4$ sec.

Tools / Approach: $a(s)\,ds = v\,dv$, determine v as a ~~fxn~~ function of s.

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Then ~~use~~ replace $v = \frac{ds}{dt}$, Find by integration $s = s(t)$. Then calculate $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$ and find $a(4)$.

$\boxed{\text{Solution}}$

$$a(s)\,ds = v\,dv \Rightarrow \int_{s_0}^{s} 3s^{-1/3}\,ds = \int_{v_0}^{v} v\,dv$$

See Correction pg 2/5a

$$3s^{2/3}\Big|_{s_0}^{s} = \frac{v^2}{2} - \frac{v_0^2}{2}$$

$$3s^{2/3} - 3s_0^{2/3} = \frac{v^2}{2} - \cancelto{0}{\frac{v_0^2}{2}}$$

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$$\left.\frac{3s^{2/3}}{2/3}\right|_{s_0}^{s} = \frac{v^2}{2} - \frac{v_0^2}{2}$$

with $v_0 = 0 \Rightarrow \quad 9s^{2/3} - 9s_0^{2/3} = v^2$

$$\frac{ds}{dt} = \sqrt{9s^{2/3} - 9s_0^{2/3}}$$

$$\int_{s_0}^{s} \frac{ds}{\sqrt{9s^{2/3} - 9s_0^{2/3}}} = \int_{t_0}^{t} dt$$

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Replace $v = \frac{ds}{dt}$

$v^2 = 6s^{2/3} - 6s_0^{2/3}$; $v = \sqrt{6s^{2/3} - 6s_0^{2/3}}$

$\cancel{\frac{ds}{dt} = 6s^{1/3} - 6s_0^{1/3}}$

$\frac{ds}{dt} = \sqrt{6s^{2/3} - 6s_0^{2/3}}$

See correction slide 2/5a

$$\frac{ds}{\sqrt{6s^{2/3} - 6s_0^{2/3}}} = dt$$

$$\cancel{\int_{s_0}^{s} \frac{ds}{\sqrt{6s^{2/3}}} = \int_{t_0}^{t} dt}$$

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See slide 2/7a for correction

$$\int_{s_0}^{s} \frac{ds}{\sqrt{6s^{2/3} - 6s_0^{2/3}}} = \int_{t_0}^{t} dt$$

Now we must assume that $s_0 = 0$ to integrate!!

$$\int_0^s \frac{ds}{\sqrt{6s^{2/3}}} = \int_0^t dt \Rightarrow \int_0^s \frac{1}{\sqrt{6}} s^{-\frac{1}{3}} ds = t$$

$$\frac{1}{\sqrt{6}} s^{2/3} \Big|_0^s = t \Rightarrow \underbrace{\frac{1}{\sqrt{6}}}_{\text{mistake}} s^{2/3} = t$$

$$\Rightarrow \quad s = (\sqrt{6}\, t)^{3/2}$$

University of Idaho with $s_0 = 0$, $t_0 = 0$

$$\frac{1}{3}\int_0^s s^{-\frac{1}{3}}\,ds = t \Rightarrow \frac{1}{3}\,\frac{1}{2/3}\,s^{2/3} = t$$

$$\frac{1}{2}s^{2/3} = t \Rightarrow s = 2^{\frac{3}{2}}t^{\frac{3}{2}}$$

$$\frac{ds}{dt} = v = \frac{3}{2}2^{\frac{3}{2}}t^{\frac{1}{2}}$$

$$\frac{dv}{dt} = a = \frac{1}{2}\,\frac{3}{2}\,2^{\frac{3}{2}}t^{-\frac{1}{2}}$$

$$\boxed{a(4) = \frac{1}{2}\,\frac{3}{2}\,2^{\frac{3}{2}}\frac{1}{2} = 1.06\ m}$$

See slide 2/7a for correction

University of Idaho Now, we can calculate $v = \frac{ds}{dt}$ & $a = \frac{ds}{dt}$ and find a at $t = 4$ sec

$\cancel{\frac{ds}{dt}}$ $\left(6^{\frac{1}{2}}\right)^{\frac{3}{2}} = 6^{3/4}$ $s = 6^{3/4} t^{3/2}$

$\frac{ds}{dt} = v = 6^{3/4} \frac{3}{2} t^{\frac{1}{2}}$

$\frac{d^2s}{dt^2} = \frac{dv}{dt} = a = 6^{3/4} \frac{3}{2} t^{-\frac{1}{2}}$ m/s²

so a at $t = 4$ sec is $\boxed{a(4) = 6^{\frac{3}{4}} \frac{3}{2} \frac{1}{2} \frac{m}{s^2}}$

Ans.

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Section 12.3 Rectilinear Kinematics: Erratic Motion:

In this section, we consider one-dimensional motion in which position, velocity, and/or acceleration are not continuous.

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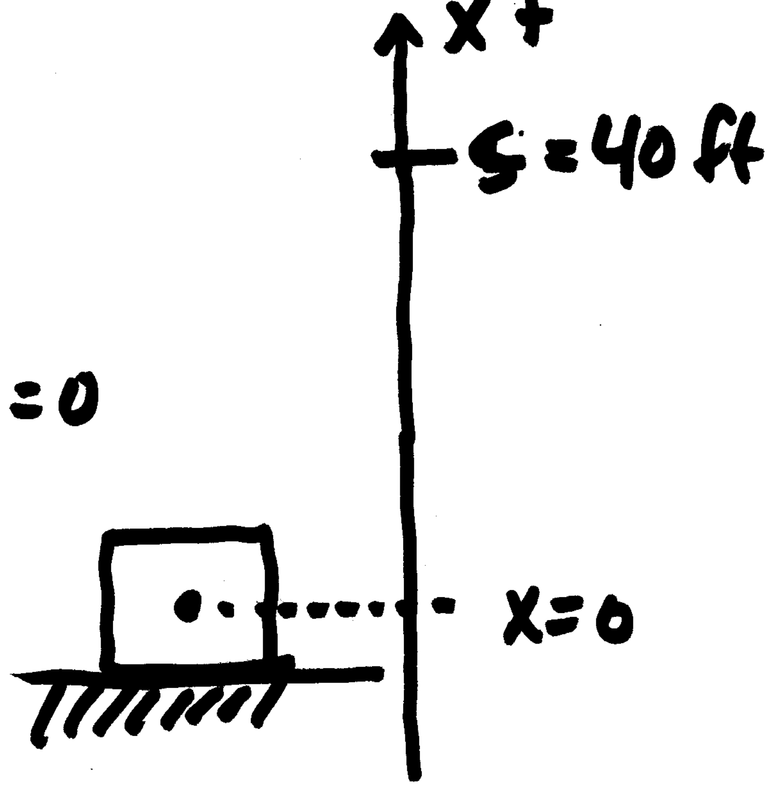
Example Problem 12-38



Given

- Elevator starts at $x=0$ from rest v at $s=0$ is $v=0$
- Elevator first accelerates at $a = 5\ ft/s^2$, then de-accelerates $a = -2\ ft/s^2$
- At $s = x = 40\ ft$, the elevator comes to rest. v at $s=40$ is $v=0$

Find · Determine shortest time for the elevator to reach 40 ft.

Tools / Approach