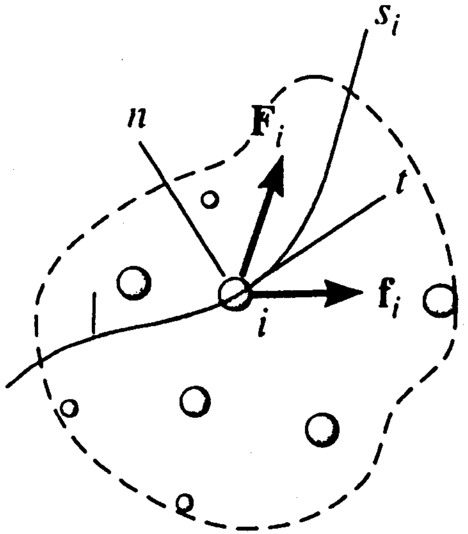
Work/Energy Principle for Systems of Ptcles



Inertial coordinate system

$$\underbrace{\frac{1}{2}\sum_i m_i v_{i1}^2}_{\sum T_{i1}} + \underbrace{\sum_i \int_{s_{i1}}^{s_{i2}} (\vec{f}_i)_t \, ds + \sum_i \int_{s_{i1}}^{s_{i2}} (\vec{F}_i)_t \, ds}_{\sum U_{i,1-2}} = \underbrace{\sum_i \frac{1}{2} m_i v_{i2}^2}_{\sum T_{i2}}$$

$$\sum T_{i1} + \sum U_{i,1-2} = \sum T_{i2}$$

$$\sum U_{i,1-2} = \begin{bmatrix} \text{Work done} \\ \text{by internal forces} \end{bmatrix} + \begin{bmatrix} \text{Work done by} \\ \text{externally appl} \\ \text{forces} \end{bmatrix}$$

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$$\begin{bmatrix}\text{Work done by} \\ \text{internal forces}\end{bmatrix} = 0 \Rightarrow \sum U_{i,1\to 2} = \sum_i \int_{s_{i1}}^{s_{i2}} (\vec{F}_i)_t \, ds$$

However, for non-rigid, or deformable bodies,

$$\begin{bmatrix}\text{Work done by} \\ \text{internal forces}\end{bmatrix} \neq 0 \Rightarrow$$ Energy is stored in the deformation of the body.

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Conservative Forces and Potential

Energy

Work/Energy Principle

$$T_1 + U_{1-2} = T_2$$

$$U_{1-2} = \underbrace{\begin{bmatrix}\text{Work done} \\ \text{by gravity} \\ \text{force}\end{bmatrix} + \begin{bmatrix}\text{Work done} \\ \text{by spring}\end{bmatrix}}_{\text{Work Caused By Conservative Forces}} + \underbrace{\begin{bmatrix}\text{Work done by} \\ \text{applied forces} \\ \text{and friction}\end{bmatrix}}_{\text{Nonconservative Work}}$$

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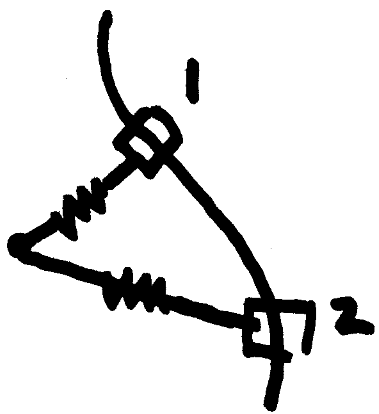
$$U_{1-2} = V + (U_{1-2})_{\text{nonconservative}}$$

• path independent, gravity $V = Wy$

depends only on altitude (height)

Spring $V = \frac{1}{2}ks^2$ ($s=0$ when spring is undeformed)



The only thing that matters is the extension (compression) at s_1 and s_2.

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$(U_{1-2})_{nonconservative}$ $\Rightarrow$

- path dependent, must calculate $\int_{s} \vec{F} \cdot d\vec{r}$
- energy is either added, or subtracted to the system i.e, friction subtracts energy

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Then define

potential energy gravity

potential energy by spring

$$U_{1-2} = V_1 - V_2 \qquad V = V_g + V_e$$

$$(U_{1-2})_{noncons} = 0 \qquad = Wy + \frac{1}{2}ks^2$$

into $T_1 + U_{1-2} = T_2$

$$T_1 + (V_1 - V_2) = T_2 \Rightarrow T_1 + V_1 = T_2 + V_2 = \text{Energy of system}$$

$$V = Wy + \frac{1}{2}ks^2$$

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$$V_1 = Wy_1 + \tfrac{1}{2}ks_1^2, \quad V_2 = Wy_2 + \tfrac{1}{2}ks_2^2$$

$$T_1 + (V_1 - V_2) = T_2$$

~~Fot~~

$$T_1 \underbrace{- W(y_2 - y_1)}_{U_{1\text{-}2}\ \text{gravity}} \underbrace{- \tfrac{1}{2}k(s_2^2 - s_1^2)}_{U_{1\text{-}2}\ \text{spring}} = T_2$$

$s = 0$ at undeformed spring.