University of Idaho **Final Exam**

- Mon Dec 13, 7:30 AM - 9:30 AM.
- B. Final Exam 4pts Covers [Ex1, Ex2, Ex3, WE : I/M]
- If an Ex "X" Score Score Corresponding ~~to Ex1,2,3~~ tow is higher than corresponding low Ex1,2,3, replace low with Fe corresponding FE component score.

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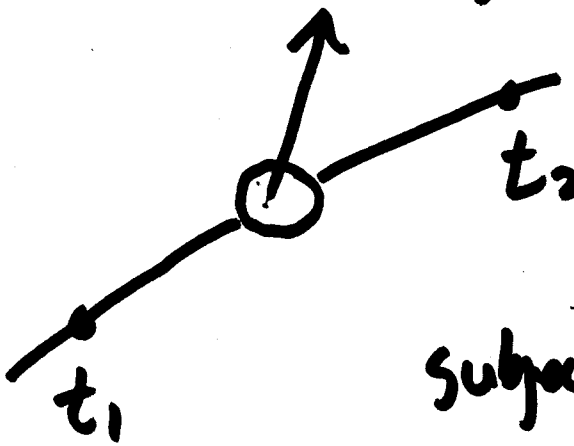
Impulse / Momentum for Particles

Sections 15.1 & 15.2 - Application to particles and systems of particles

Say for a particle

$\vec{F}(t)$ = Net external force = $\sum_i \vec{F}_i$



During the motion, it is subject to Newton's 2nd law

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

University of Idaho In kinetics - we applied Newton's 2nd Law to determine properties of motion at an instant.

Now we will integrate Newton's 2nd law, and relate properties at $t=t_1$ to $t=t_2$ by the integrated external force

$$\vec{F} = m\frac{d\vec{v}}{dt} \Rightarrow \int_{t_1}^{t_2} \vec{F}\,dt = \int_{t_1}^{t_2} m\frac{d\vec{v}}{dt}\,dt$$

$$\int_{t_1}^{t_2} \vec{F}(t)\,dt = m\vec{v}(t_2) - m\vec{v}(t_1) = m\vec{v}_2 - m\vec{v}_1$$

$m\vec{v}_2$: velocity @ $t=t_2$

$m\vec{v}_1$: velocity @ $t=t_1$

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$$m\vec{v}_1 + \underbrace{\int_{t_1}^{t_2} \vec{F}(t)\,dt}_{\text{Impulse} \sim kg\,\frac{m}{s}} = m\vec{v}_2$$

This is a vector equation, we can split into three scalar equations,

$$\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}, \quad \vec{F}(t) = F_x(t)\vec{i} + F_y(t)\vec{j} + F_z(t)\vec{k}$$

$$mv_{x1}\vec{i} + mv_{y1}\vec{j} + mv_{z1}\vec{k} + \int_{t_1}^{t_2} F_x(t)\,dt\,\vec{i} + \int_{t_1}^{t_2} F_y(t)\,dt\,\vec{j} + \int_{t_1}^{t_2} F_z(t)\,dt\,\vec{k} = mv_{x2}\vec{i} + mv_{y2}\vec{j} + mv_{z2}\vec{k}$$

University of Idaho Equating coeffs of $\vec{i}, \vec{j}, \vec{k}$:

$$m v_{x1} + \int_{t_1}^{t_2} F_x(t)\,dt = m v_{x2}$$

$$m v_{y1} + \int_{t_1}^{t_2} F_y(t)\,dt = m v_{y2}$$

$$m v_{z1} + \int_{t_1}^{t_2} F_z(t)\,dt = m v_{z2}$$

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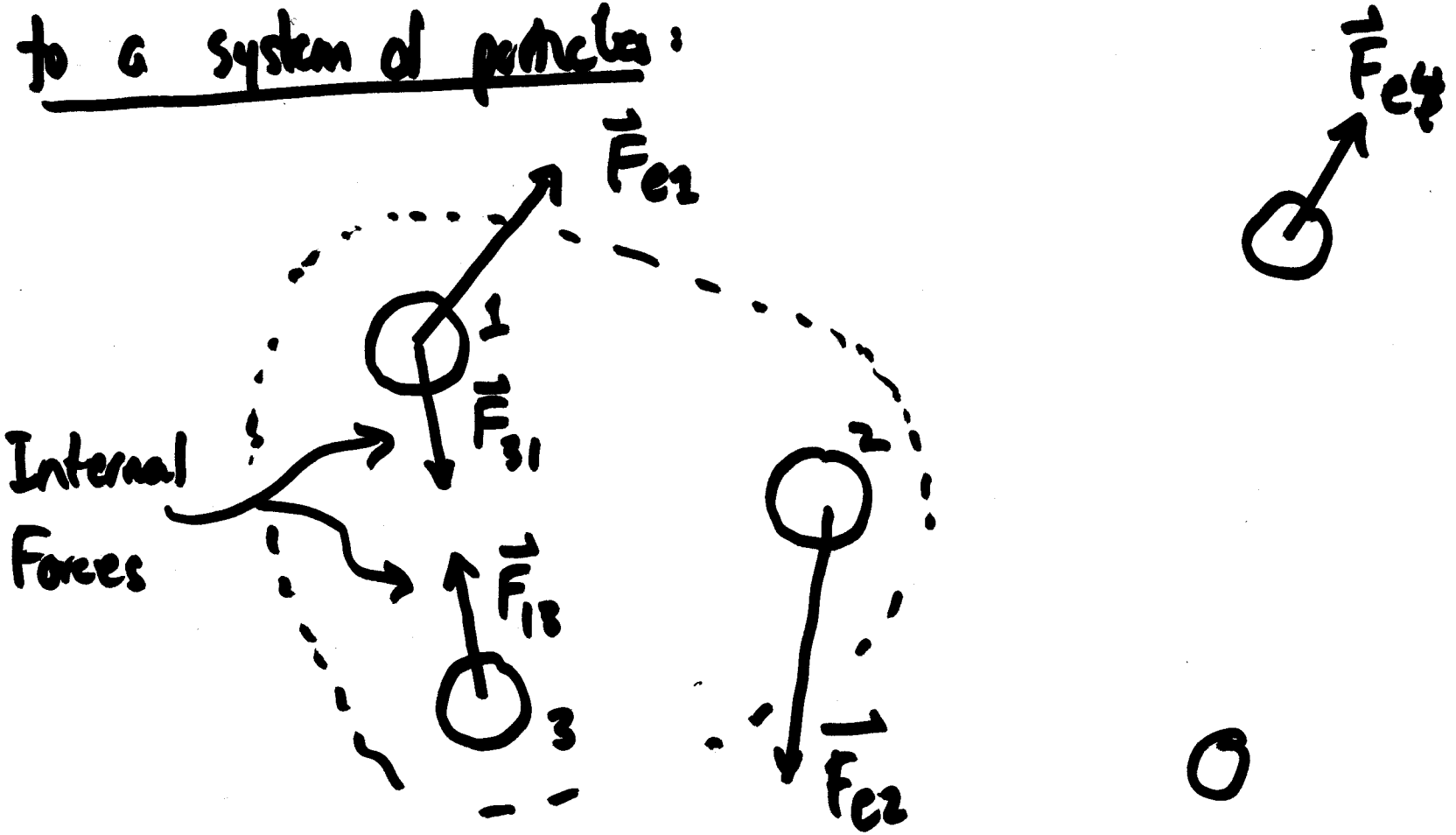
An IM principle can be applied to a system of particles:



University of Idaho For the system of particles i ($i=1,2,3$), the impulse momentum principle is

$$\sum_i m_i \vec{v}_{i1} + \int_{t_1}^{t_2} \sum \vec{F}_e(t)\,dt = \sum m_i \vec{v}_{i2}$$

- $\vec{v}_{i1}$: velocity of each ptcle at $t=t_1$
- $\int_{t_1}^{t_2} \sum \vec{F}_e(t)\,dt$: sum of externally applied forces
- $\vec{v}_{i2}$: velocity velocity of each ptcle at $t=t_2$

m_i = mass of ptcle i.

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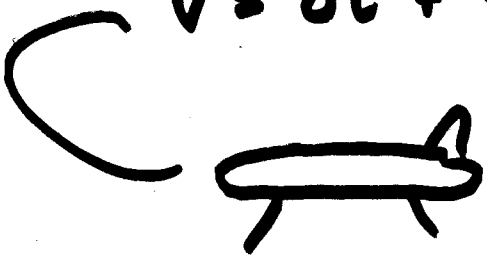
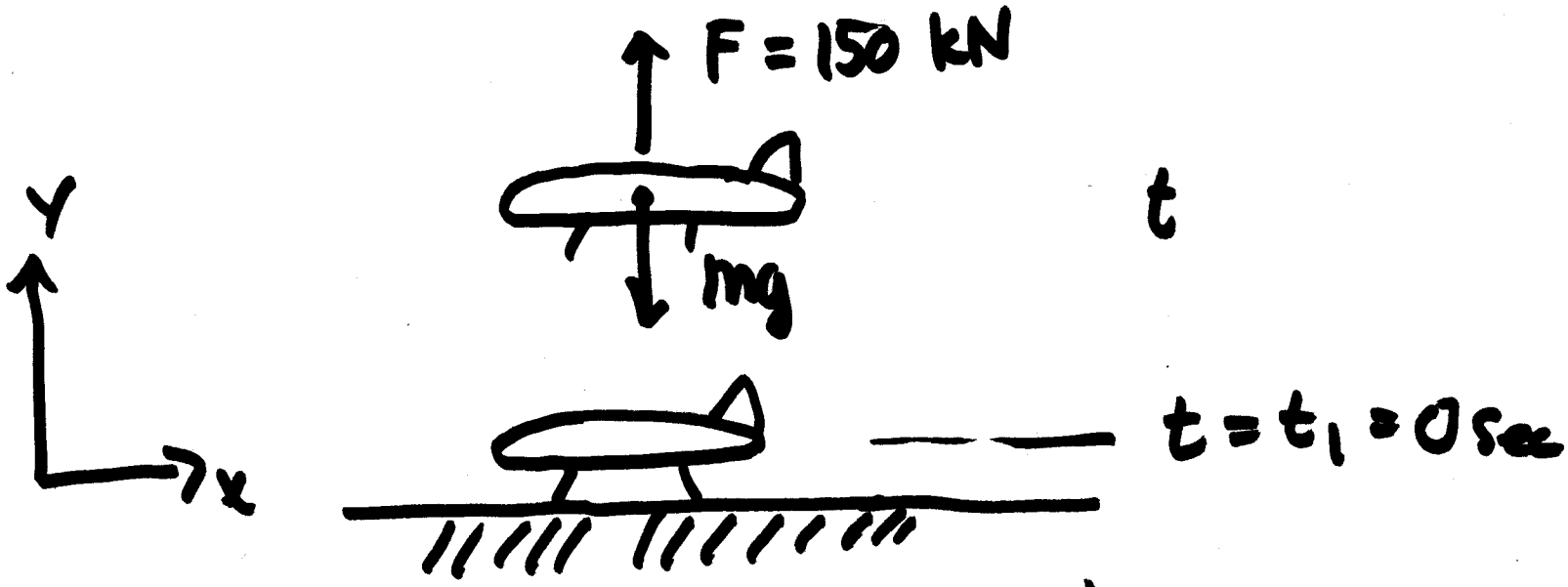
$\vec{v} = 0\vec{i} + v_{y_2}\vec{j} + 0\vec{k}$



$t = t_2 = 6\,sec$



$\vec{v}_1 = 0\vec{i} + 0\vec{j} + 0\vec{k}$