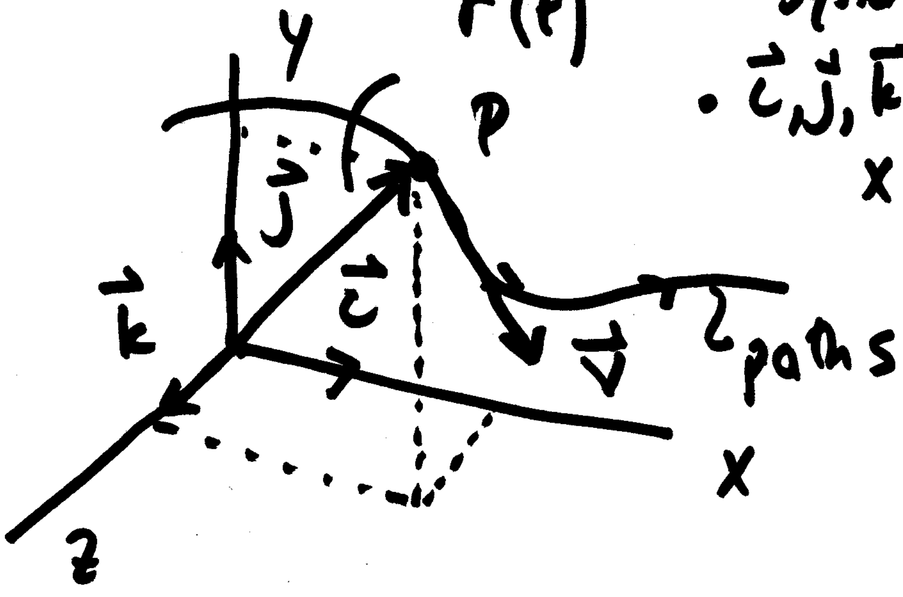
University of Idaho **Sections 12.4, 12.5, 12.6**

General Curvilinear Motion, Rectangular Components, Projectile Motion

General Curvilinear Motion



- xyz - **fixed** coordinate system
- $\vec{i}, \vec{j}, \vec{k}$ are unit vectors in x, y, z directions

University of Idaho $\vec{r}(t)$ = position vector.

$$\vec{r}(t) = x(t)\,\vec{i} + y(t)\,\vec{j} + z(t)\,\vec{k}$$

$$|\vec{r}| = \sqrt{x^2+y^2+z^2}$$

Velocity vector $\vec{v} = \frac{d\vec{r}}{dt}$

$$= \frac{d}{dt}(x\vec{i}) + \frac{d}{dt}(y\vec{j}) + \frac{d}{dt}(z\vec{k})$$

Since $\vec{i}, \vec{j}, \vec{k}$ are fixed

$$\vec{v} = \dot{x}\,\vec{i} + \dot{y}\,\vec{j} + \dot{z}\,\vec{k} \qquad \dot{} = \frac{d}{dt}$$

The velocity vector is instantaneously tangent to the path.

Magnitude of the velocity vector

$$|\vec{v}| = v = \text{speed} = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \quad \text{scalor}$$

Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}$

In general, not tangent to path.

Alternate notation

$$\vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

$$v_x = \dot{x}, \quad v_y = \dot{y}, \quad v_z = \dot{z}$$

University of Idaho Same for the acceleration

$$\vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

Application of the chain rule for time differentiation...

Suppose we have a function

$$y(t) = f[g(t)] \qquad y(t) = 13g^2, \quad g = 49t$$

$$\frac{dy}{dt} = \frac{dy}{dg}\frac{dg}{dt} \text{ (chain rule).}$$

e.g. $\frac{dy}{dt} = 26g \cdot 49 = 26 \cdot 49 \cdot 49t$

Another, more common example

$y = \sin\theta$, where $\theta = \theta(t)$

$$\frac{dy}{dt} = [\cos\theta]\,\dot{\theta} = \dot{\theta}\cos\theta$$