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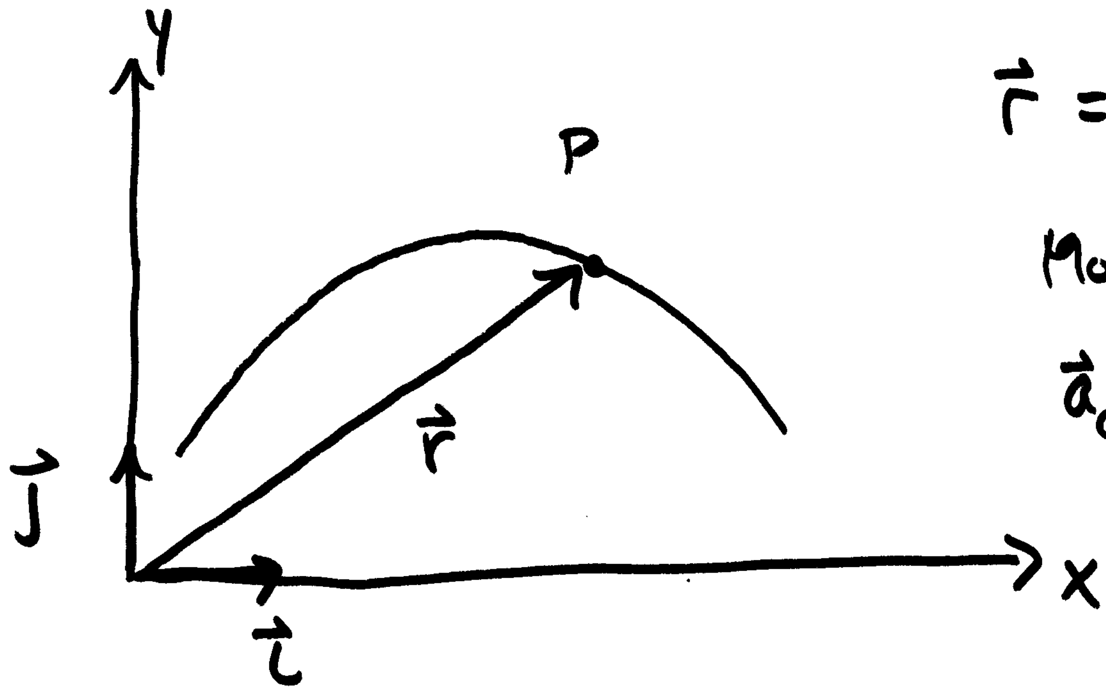
Motion of a Projectile



$\vec{r} = x\vec{i} + y\vec{j}$

Motion is such that

$\vec{a}_c = \cancel{\vec{g}} = -g\vec{j}$

$g = 9.81 \, m/s^2$

$= 32.2 \, ft/s^2$

We want to determine $\vec{r}$, & $\vec{v}$ from $\vec{a}$. We do this by integration.

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$$\vec{a}_c = -g\vec{j} = \ddot{x}\,\vec{i} + \ddot{y}\,\vec{j} = \dot{v}_x\,\vec{i} + \dot{v}_y\,\vec{j}$$

Horizontal motion

$$\vec{a}_c = 0\vec{i} - g\vec{j} = \dot{v}_x\vec{i} + \dot{v}_y\vec{j}$$

$$\frac{dv_x}{dt} = 0 \Rightarrow \boxed{v_x(t) = \text{constant} = (V_0)_x}$$

$$\vec{r} = x\vec{i} + y\vec{j} \qquad \vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j} = v_x\vec{i} + v_y\vec{j}$$

$$\frac{dx}{dt} = v_x(t) \Rightarrow \int_{x_0}^{x} dx = \int_{V_0}^{V} \cancel{v_x}(t) = \int_{t_0}^{t} v_x(t)\,dt$$

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$$x - x_0 = \int_{t_0}^{t} (v_0)_x \, dt = (v_0)_x (t - t_0)$$

$$\boxed{x = x_0 + (v_0)_x (t - t_0)}$$ often $t_0 = 0$

<u>Vertical Motion</u>

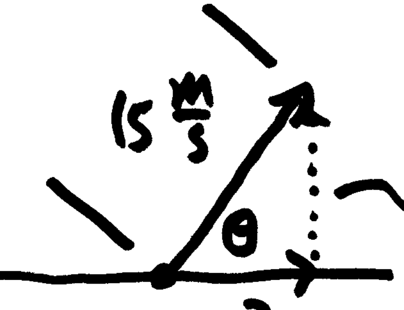
By equating components $\Rightarrow$ $\dfrac{dv_y}{dt} = -g$

$$\int_{(v_0)_y}^{v} dv_y = \int_{t_0}^{t} -g\,dt \Rightarrow v_y - (v_0)_y = -g(t - t_0)$$

$$\boxed{v_y = (v_0)_y - g(t - t_0) \quad t_0 = 0}$$

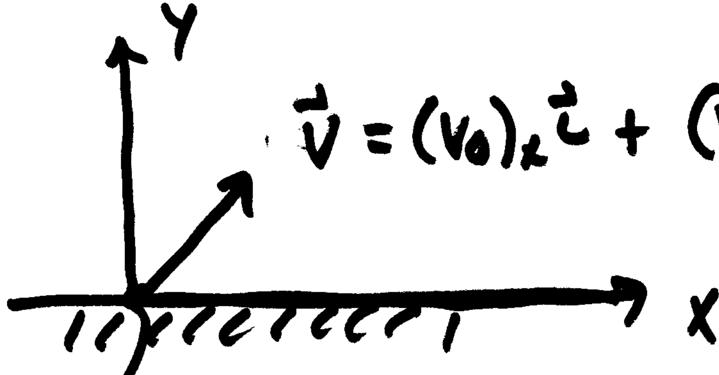
$\vec{v} = (v_0)_x \vec{i} + (v_0)_y \vec{j}$

$= \text{velocity vector at } t = t_0 = 0$



$(v_0)_y \vec{j}$

$(v_0)_x \vec{i}$



$\vec{v} = (v_0)_x \vec{i} + (v_0)_y \vec{j}$

$(x_0, y_0) = (0, 0)$

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$$\frac{dy}{dt} = v_y(t) \Rightarrow \int_{y_0}^{y} dy = \int_{t_0}^{t} v_y(t)\,dt$$

$$= \int_{t_0}^{t} \left[(v_0)_y - g(t-t_0)^{\cancel{x}} \right] dt$$

$$y - y_0 = (v_0)_y (t-t_0) - \frac{1}{2} g (t-t_0)^2$$

$$\boxed{y = y_0 + (v_0)_y (t-t_0) - \frac{1}{2} g (t-t_0)^2 \quad t_0 = 0}$$

4/5

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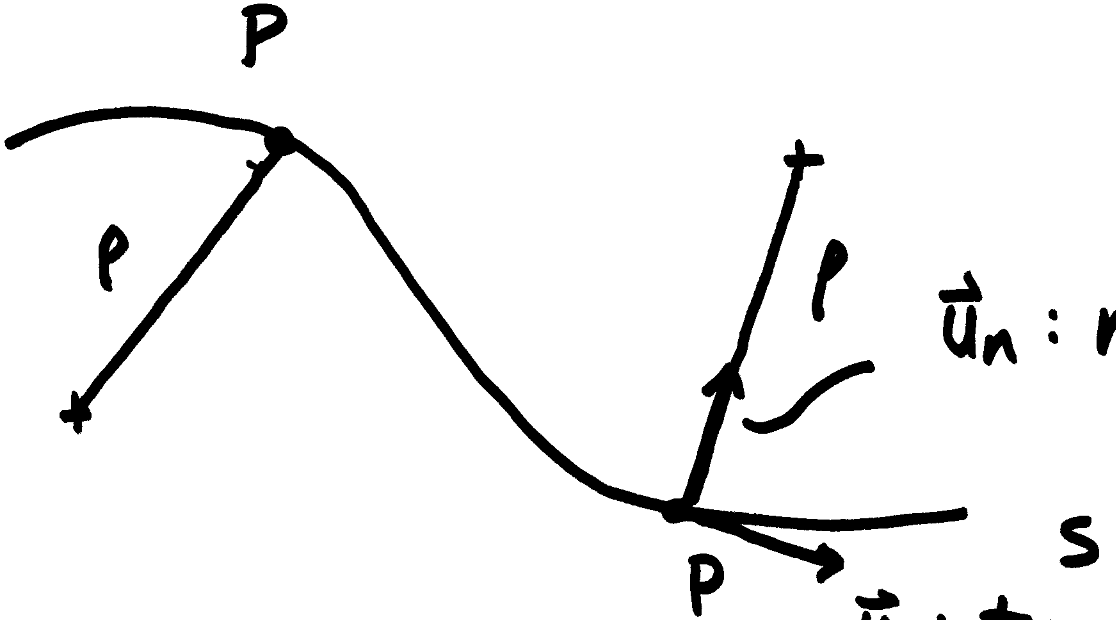
Section 12.7 Curvilinear Motion, Normal & Tangential Coordinates

Case of Planar Motion

* $\vec{u}_n, \vec{u}_t$ lie in the osculating plane



$\vec{u}_n$: normal direction pts toward center of curvature

$\vec{u}_t$: tangent direction, points in direction of increasing s.

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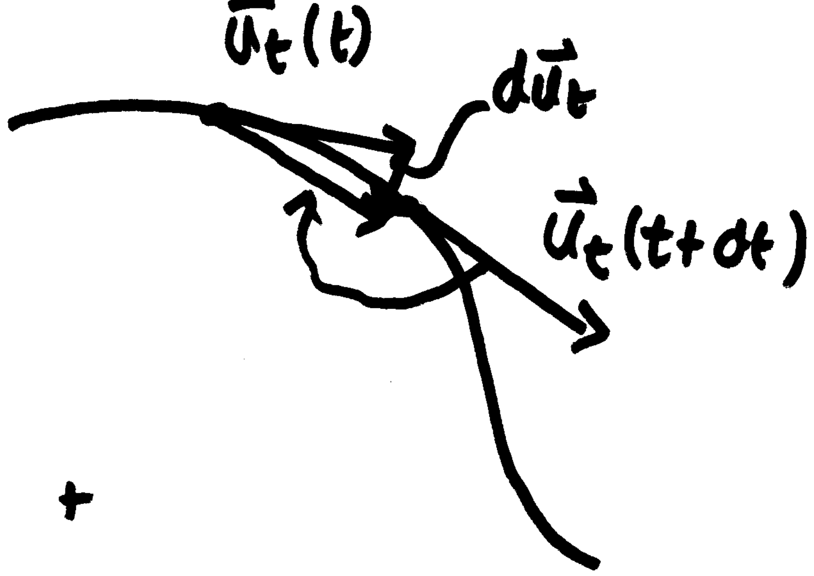
Again, we're interesting in relating position, velocity and acceleration.

Velocity; $\vec{v} = v\,\vec{u}_t$

Acceleration; $\vec{a} = \dfrac{d\vec{v}}{dt} = \dot{v}\,\vec{u}_t + v\,\dfrac{d\vec{u}_t}{dt}$

direction of $\vec{u}_t$ changes with time.



$\vec{u}_t(t+dt) = \vec{u}_t(t) + d\vec{u}_t$

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$$\frac{d\vec{u}_t}{dt} = \frac{v}{\rho}\vec{u}_n$$

$$\vec{a} = \dot{v}\vec{u}_t + \frac{v^2}{\rho}\vec{u}_n$$

$$= a_t\vec{u}_t + a_n\vec{u}_n$$

normal acceleration $a_n = \frac{v^2}{\rho}$

tangential acceleration

$$a_t = \dot{v}$$

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We can "steal" a set of tools for special case of constant tangential acceleration;

$a_{t_c} = \dot{v} = \text{constant}$

$v = v_0 + a_{t_c} t$

Constant acceleration along a straight line: $\underline{\underline{a_c}}$

$$v = v_0 + a_c t$$

$$x = x_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

$$v = v_0^2 + 2a_c (x - x_0)$$

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$$s = s_0 + v_0 t + \frac{1}{2} a_{ct} t^2$$

$$v = v_0^2 + 2 a_{tc} (s - s_0)$$