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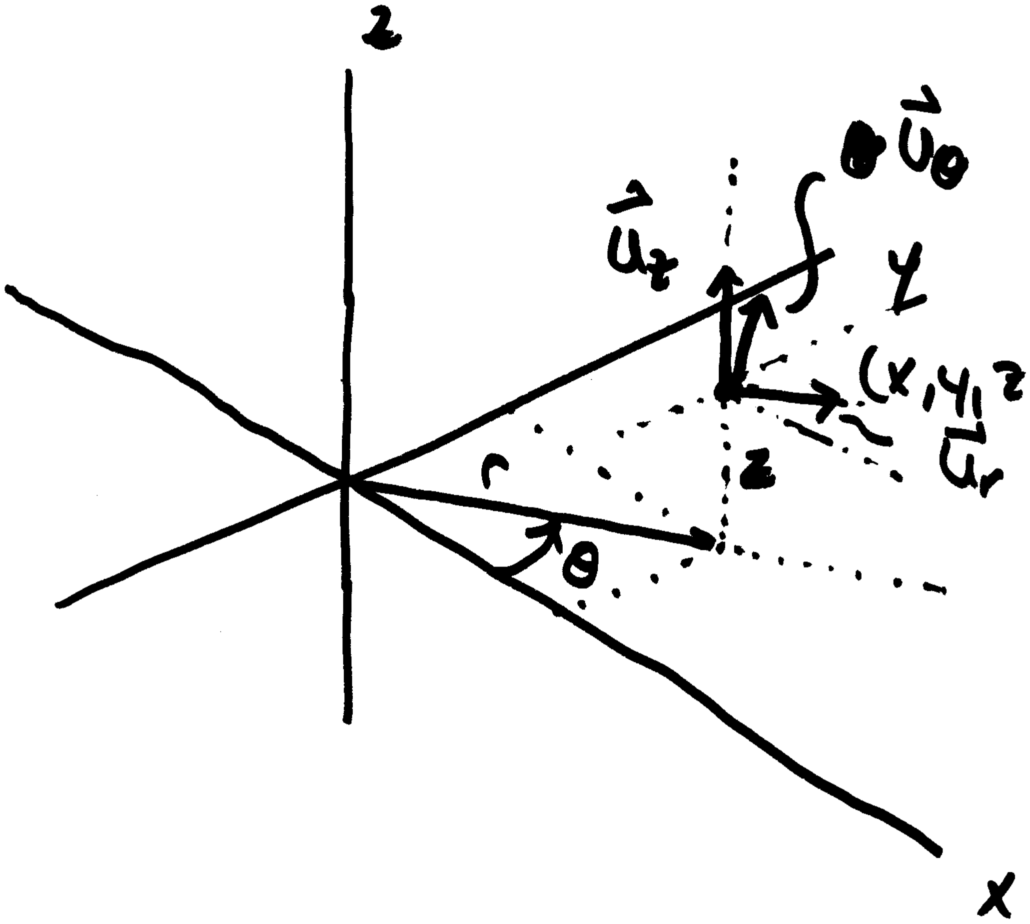
Section 12.8 - Curvilinear Motion - Radial & Cylindrical Components



xyz - fixed coordinate system

$(x, y, z) = (r, \theta, z)$

$\vec{u}_r \times \vec{u}_\theta = \vec{u}_z$

By right-hand rule

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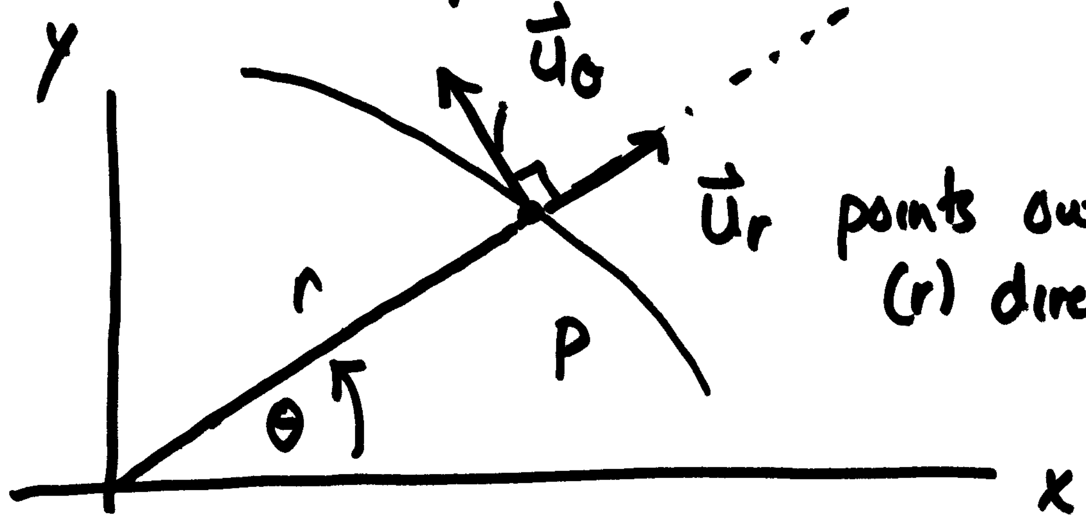
Consider Motion in (x,y) or (r,θ) plane $(z=0)$ Top View

xy is fixed



$\vec{u}_r$ points outward in radial (r) direction

$\vec{u}_\theta$: $\perp$ to $\vec{u}_r$ & points in direction of increasing θ.

$\Rightarrow$ Cylindrical coord system moves with the ptcle.

$\Rightarrow$ $\vec{u}_r$ nn $\perp$ to path, & $\vec{u}_\theta$ nn tangent to path.

Determine relationships between position, velocity and acceleration (kinematics):

$$\vec{r} = r\vec{u}_r \quad (ft, m)$$

In 3-D $\vec{r} = r\vec{u}_r + z\vec{u}_z$

Velocity $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\vec{u}_r) = \dot{r}\vec{u}_r + r\frac{d(\vec{u}_r)}{dt}$

↑ accounts for $\vec{u}_r$ changing direction with time

$$\frac{d(\vec{u}_r)}{dt} = \dot{\theta}u_\theta$$

$$\boxed{\vec{v} = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta}$$

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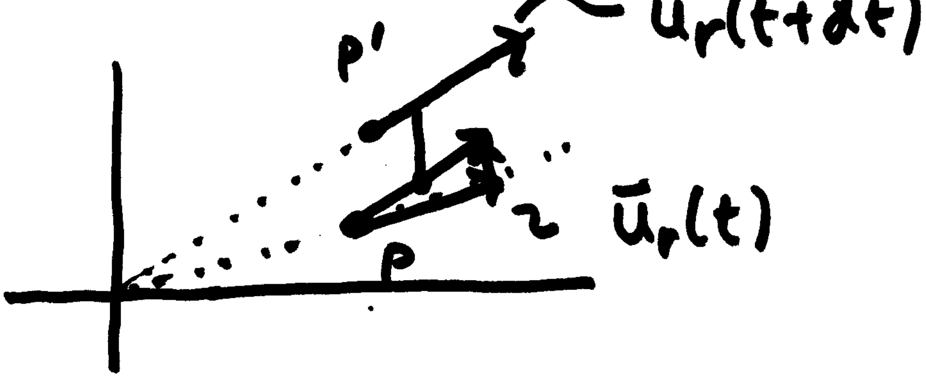
Geometry of the motion



$$\frac{d\vec{u}_r}{dt} = \frac{\vec{u}_r(t+dt) - \vec{u}_r(t)}{dt}$$

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Acceleration vector;

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{r}\,\frac{d(\vec{u}_r)}{dt} + \ddot{r}\,\vec{u}_r + \frac{d}{dt}(r\dot{\theta})\vec{u}_\theta + r\dot{\theta}\,\frac{d(\vec{u}_\theta)}{dt}$$

$$= \dot{r}\dot{\theta}\,\vec{u}_\theta + \ddot{r}\,\vec{u}_r + r\ddot{\theta}\,\vec{u}_\theta + \dot{r}\dot{\theta}\,\vec{u}_\theta + r\dot{\theta}(-\dot{\theta}\,\vec{u}_r)$$

Some vector Δ as for n-t system

$$\boxed{\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta}$$

<u>In 3-D</u>

$$\vec{r} = r\,\vec{u}_r + z\,\vec{u}_z$$

$$\vec{v} = \dot{r}\,\vec{u}_r + r\dot{\theta}\,\vec{u}_\theta + \dot{z}\,\vec{u}_z$$

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$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta + \ddot{z}\,\vec{u}_z$$