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Section 12.9 Absolute Dependent Motion Analysis of Two Particles

⇒ Motion of one particle directly determines the motion of other particles.

⇒ Pulley problems

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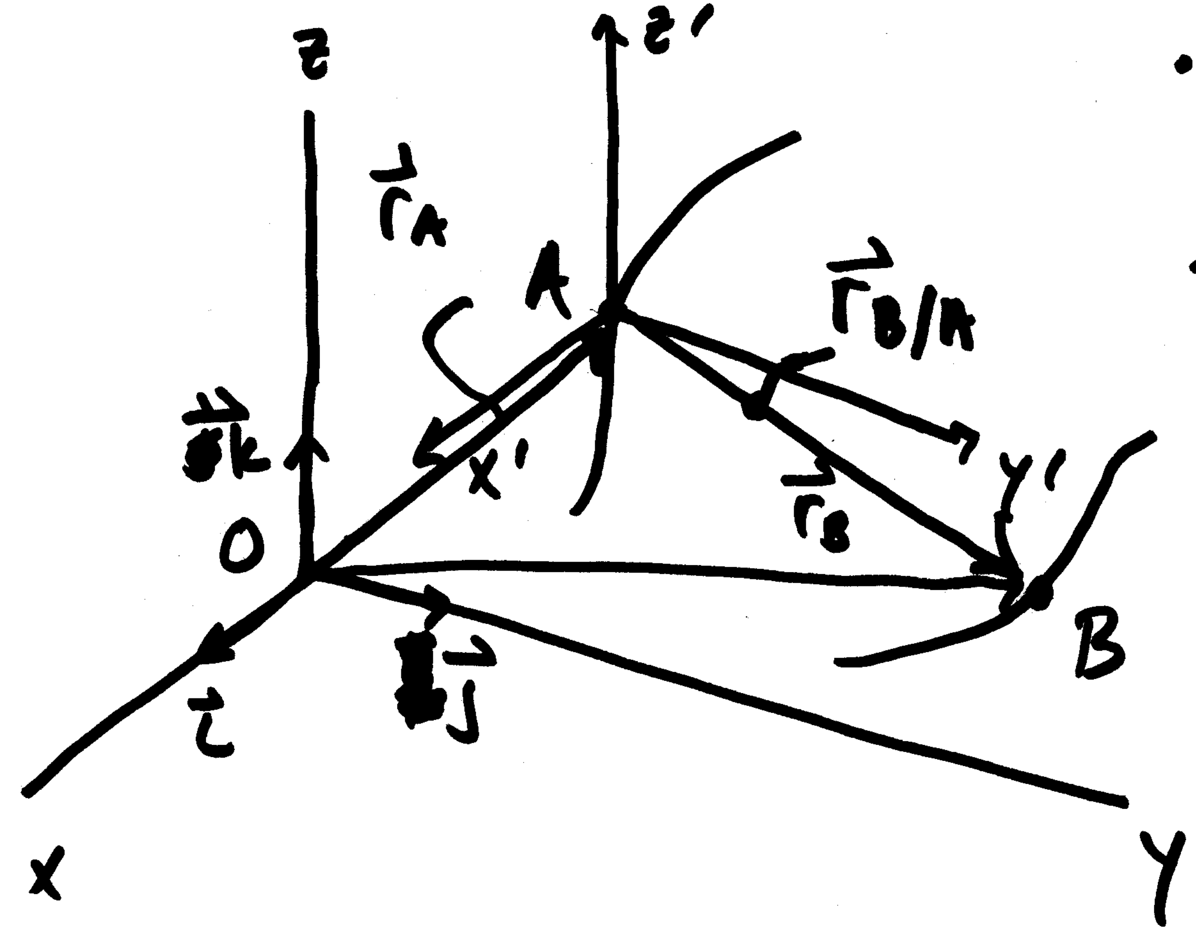
Section 12.10 Relative Motion of Two Particles Using Translating Axes



- xyz fixed in space
- $\vec{i}, \vec{j}, \vec{k}$ attached to xyz
- Frame $x'y'z'$ attached to particle A
- $x'y'z' \parallel xyz$

  $x'y'z'$ translates but does not rotate

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- $\vec{i}', \vec{j}', \vec{k}'$ to the system $x'y'z'$
- Because $x'y'z'$ translates, but does not rotate

  $\vec{i}' = \vec{i}$, $\vec{j}' = \vec{j}$, $\vec{k}' = \vec{k}$

$$\left.\begin{aligned} \vec{r}_A &= x_A\vec{i} + y_A\vec{j} + z_A\vec{k} \\ \vec{r}_B &= x_B\vec{i} + y_B\vec{j} + z_B\vec{k} \end{aligned}\right\} \text{Position of A \& B relative to the fixed frame } xyz$$

$$\left.\begin{aligned} \vec{r}_{B/A} &= x'_{B/A}\vec{i}' + y'_{B/A}\vec{j}' + z'_{B/A}\vec{k}' \\ &= x'_{B/A}\vec{i} + y'_{B/A}\vec{j} + z'_{B/A}\vec{k} \end{aligned}\right\} \text{Position of B as it appears from A}$$

University of Idaho Supposing that we know two of the three $\vec{r}_A$, $\vec{r}_B$, $\vec{r}_{B/A}$, and wish to calculate the other, we can use the vector ~~trang~~ triangle equation

$$\cancel{\vec{r}_{B/A}} \quad \boxed{\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}}$$

We can easily change, place the $x'y'z'$ system on B, and get

$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

University of Idaho We can extend to relative velocity by th taking the time derivative:

$$\frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{r}_{B/A}}{dt} \Rightarrow \boxed{\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}}$$

$$\wr$$

$$\frac{d\vec{r}_B}{dt} = \frac{d}{dt}\left(x_B\vec{i}\right) + \frac{d}{dt}\left(y_B\vec{j}\right) + \frac{d}{dt}\left(z_B\vec{k}\right)$$

$$= \dot{x}_B\vec{i} + \dot{y}_B\vec{j} + \dot{z}_B\vec{k}$$

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$$\dot{x}_B\vec{i} + \dot{y}_B\vec{j} + \dot{z}_B\vec{k} = \dot{x}_A\vec{i} + \dot{y}_A\vec{j} + \dot{z}_A\vec{k} + \dot{x}'_{B/A}\vec{i} + \dot{y}'_{B/A}\vec{j} + \dot{z}'_{B/A}\vec{k}$$

And, we can further extend to acceleration by taking the time derivative:

$$\cancel{\frac{d\vec{v}_A}{dt} = \frac{d\vec{v}_B}{dt} + \frac{d\vec{v}_{B/A}}{dt} \Rightarrow \vec{a}_A = a_B}$$

$$\frac{d\vec{v}_B}{dt} = \frac{d\vec{v}_A}{dt} + \frac{d\vec{v}_{B/A}}{dt} \Rightarrow \boxed{\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}}$$