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Section 13.5 Equations of Motion: Normal and Tangential Coordinates

$$\vec{F} = m\vec{a} \quad \text{2D problems}$$

$$\vec{a} = a_t \vec{u}_t + a_n \vec{u}_n = \dot{v}\vec{u}_t + \cancel{a_n}\frac{v^2}{\rho}\vec{u}_n$$

$$\vec{F} = F_n \vec{u}_n + F_t \vec{u}_t$$

$$F_n \vec{u}_n + F_t \vec{u}_t = m\left(\dot{v}\vec{u}_t + \frac{v^2}{\rho}\vec{u}_n\right)$$

University of Idaho We'll end up with the two scalar equations

$$F_t = m\dot{v} \quad \& \quad F_n = m\frac{v^2}{\rho}$$

If we have forces in the vertical direction, then we would have a third scalar equation.

Section 13.6 Equations of Motion: Cylindrical Coordinates

$$\vec{a} = a_r\vec{u}_r + a_\theta\vec{u}_\theta + a_z\vec{u}_z$$

$$= (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{u}_\theta + \ddot{z}\,\vec{u}_z$$

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The vector sum of applied external forces $\vec{F}$ will be decomposed into components along the $\vec{u}_r, \vec{u}_\theta$: $\vec{u}_z$ directions:

$$\vec{F} = F_r \vec{u}_r + F_\theta \vec{u}_\theta + F_z \vec{u}_z$$

Equating coefficients of unit vectors;

$$\boxed{\begin{aligned} F_r &= m(\ddot{r} - r\dot{\theta}^2) \\ F_\theta &= m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \\ F_z &= m\ddot{z} \end{aligned}}$$