ME413/513/504ST & WSU ME523 Engineering Acoustics/Special Topics
Exam II Review

To Memorize:

- Bessel functions look like damped sinusoids.
- $J_n(x)$ is a n^{th} order cylindrical Bessel function of the first kind.
- $Y_n(x)$ is a n^{th} order cylindrical Bessel function of the second kind.
- As $x \to 0$, $Y_n(x) \to -\infty$.
- $H_n^{(2)}(x)$ is a n^{th} order type 2 cylindrical Hankel function that represents outgoing waves, assuming a temporal dependence of $e^{j\omega t}$.
- $j_n(x)$ is a n^{th} order spherical Bessel function of the first kind.
- $y_n(x)$ is a n^{th} order spherical Bessel function of the second kind.
- As $x \to 0$, $y_n(x) \to -\infty$.
- $h_n^{(2)}(x)$ is a n^{th} order type 2 spherical Hankel function that represents outgoing waves, assuming a temporal dependence of $e^{j\omega t}$.
- $P_n^m(x)$ is a n^{th} order, m index Lagendre function. Lagendre functions are polynomials.
- As x becomes large, hn(2)(x)→$1/\sqrt{x}$

Reference

- Reflection/Transmission Coefficients, for $\hat{p}_i e^{j(\omega t - k_1 \cos\theta_i x - k_1 \sin\theta\, y)}$,
  $\hat{p}_r e^{j(\omega t + k_1 \cos\theta_r x - k_1 \sin\theta_r y)}$, $\hat{p}_t e^{j(\omega t - k_2 \cos\theta_{ti} x - k_2 \sin\theta_t y)_1}$
  $$\hat{R} = \frac{\hat{p}_r}{\hat{p}_i}, \quad \hat{T} = \frac{\hat{p}_t}{\hat{p}_i}$$
- Snell's Law $\theta_i = \theta_r$, $\sin\theta_i / \sin\theta_t = c_1/c_2$.
- Reflection/transmission coefficient for oblique incidence

$$\hat{R} = \frac{\dfrac{\rho_2 c_2}{\rho_1 c_1} - \dfrac{\cos\theta_t}{\cos\theta_i}}{\dfrac{\rho_2 c_2}{\rho_1 c_1} + \dfrac{\cos\theta_t}{\cos\theta_i}}, \quad \hat{T} = \frac{2\dfrac{\rho_2 c_2}{\rho_1 c_1}}{\dfrac{\rho_2 c_2}{\rho_1 c_1} + \dfrac{\cos\theta_t}{\cos\theta_i}}$$

If $\theta_i < \theta_c$, where $\theta_c = \sin^{-1}\left[\dfrac{c_1}{c_2}\right]$ and $c_2 > c_1$, then $\cos\theta_t = \sqrt{1 - \left(\dfrac{c_2}{c_1}\right)^2 \sin^2\theta_i}$, else $\cos\theta_t = j\sqrt{\left(\dfrac{c_2}{c_1}\right)^2 \sin^2\theta_i - 1}$

- Fourier series: $f(t) = \sum_{n=1}^{\infty}\left[C_n e^{-jn\omega t} + C_n^* e^{jn\omega t}\right]$, $\omega = 2\pi/T$, $C_0 = \dfrac{1}{2T}\int_0^T f(t)e^{jn\omega t}dt$, $C_n = \dfrac{1}{T}\int_0^T f(t)e^{jn\omega t}dt$, $n > 0$.

- Cylindrical radiation problems (r,θ,z) dependence:

$$\hat{p}(r,\theta,z) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} A_m H_m^{(2)}(k_{rn}r)\left[C_m e^{-jm\theta} + D_m e^{jm\theta}\right]\left[E_n e^{-jk_{zn}z} + F_n e^{jk_{zn}z}\right],$$

$$\hat{u}_r(r,\theta,z) = -\frac{1}{j\omega\rho_o}\sum_{n=0}^{\infty}\begin{cases} -A_o k_{rn} H_1^{(2)}(k_{rn}r)\left[C_o + D_o\right]\left[E_n e^{-jk_{zn}z} + F_n e^{jk_{zn}z}\right] \\ \quad + \sum_{m=1}^{\infty} A_m \dfrac{k_{rn}}{2}\left[H_{m-1}^{(2)}(k_{rn}r) - H_{m+1}^{(2)}(k_{rn}r)\right]\left[C_m e^{-jm\theta} + D_m e^{jm\theta}\right]\left[E_n e^{-jk_{zn}z} + \right. \end{cases}$$

$k_{rn} = \sqrt{k^2 - k_{zn}^2}$, $k = \omega/c$. A_m, C_m, D_m, E_n, F_n, $k_{zn} = n\dfrac{2\pi}{\ell_z}$ set by boundary conditions on radial velocity.

- Cylindrical radiation problems (r,θ) dependence: $E_0 = F_0 = 0.5 + 0j$, $E_n = F_n = 0$, $n > 0$, $k_{r0} = k$,

$$\hat{p}(r,\theta) = \sum_{m-0}^{\infty} A_m H_m^{(2)}(kr)\left[C_m e^{-jm\theta} + D_m e^{jm\theta}\right],$$

$$\hat{u}_r(r,\theta) = \frac{kA_0}{j\rho_o\omega} H_1^{(2)}(kr) - \frac{1}{j\rho_o\omega}\sum_{m-1}^{\infty}\frac{kA_m}{2}\left[H_{m-1}^{(2)}(kr) - H_{m+1}^{(2)}(kr)\right]\left[C_m e^{-jm\theta} + D_m e^{jm\theta}\right]$$

A_m, C_m, and D_m set by boundary conditions on radial velocity

- Cylindrical radiation problems (r) dependence: $k_{r0} = k$,

$$\hat{p}(r) = A_0 H_0^{(2)}(kr), \quad \hat{u}_r(r) = \frac{kA_0}{j\rho_o\omega} H_1^{(2)}(kr)$$

A_0 set by boundary conditions on radial velocity.

- Spherical radiation problems (r,θ,ϕ) dependence

$$\hat{p}(r,\theta,\phi) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_n h_n^{(2)}(kr) P_n^m(\cos\theta)\left[C_m e^{-jm\phi} + D_m e^{jm\phi}\right]$$

$$\hat{u}_r(r,\theta,\phi) = -\frac{1}{\rho_o j\omega}\sum_{m=0}^{\infty}\left(\begin{array}{l} -A_0 k h_1^{(2)}(kr)\left[C_0 + D_0\right] + \\ \sum_{n=1}^{\infty}\frac{A_n k}{2n+1}\left[n h_{n-1}^{(2)}(kr) - (n+1)h_{n+1}^{(2)}(kr)\right]P_n^m(\cos\theta)\left[C_m e^{-jm\phi} + D_m e^{jm\phi}\right]\end{array}\right)$$

A_n, C_m, D_m determined by boundary conditions on radial velocity.

- Spherical radiation problems (r,θ) dependence C_0=0.5+0j, D_0=C_0*, C_m=D_m=0 for m>0

$$\hat{p}(r,\theta) = \sum_{n=0}^{\infty} A_n h_n^{(2)}(kr) P_n(\cos\theta)$$

$$\hat{u}_r(r,\theta) = -\frac{1}{\rho_o j\omega}\left\{\begin{array}{l} -A_0 k h_1^{(2)}(kr) + \\ \sum_{n=1}^{\infty}\frac{A_n k}{2n+1}\left[n h_{n-1}^{(2)}(kr) - (n+1)h_{n+1}^{(2)}(kr)\right]P_n(\cos\theta)\end{array}\right\}$$

A_n determined by boundary conditions on radial velocity.

- Spherical radiation problems (r) dependence

$$\hat{p}(r) = A_0 h_0^{(2)}(kr)$$

$$\hat{u}_r(r) = \frac{A_0 k}{\rho_o j\omega} h_1^{(2)}(kr)$$

A_0 determined by boundary conditions on radial velocity.

- Lagendre series for θ-dependent boundary conditions determined by an arbitrary function f(θ) $f(\theta) = \sum_{n=1}^{\infty} A_n P_n(\cos\theta)$, $A_n = \frac{2n+1}{2}\int_0^{\pi} P_n(\cos\theta) f(\theta)\sin\theta\, d\theta$