

ME413/ME513/ME504ST/ME523/EE579 HW#1

Fundamentals of Acoustics in Fluids

1. Given the following examples of a complex number X , compute $\arg[X]$, $\operatorname{Re}[X]$, $\operatorname{Im}[X]$, $|X|$, X^* . Specify $\arg[X]$ as defined by a domain beginning at zero on the Re axis, and increasing in positively in a counterclockwise direction until 2π is reached at a full revolution. Also report $\arg[X]$ as would be reported by the “arctan” function on your calculator.
 - a. $X=1+0j$
 - b. $X=-1+0j$.
 - c. $X=1+2j$.
 - d. $X=-2+1j$
 - e. $X=-1-3j$.

2. Given that $X=a+jb$, $Y=c+jd$, and $\phi_x=\arg[X]$, $\phi_y=\arg[Y]$, show that
 - a. $|XY|=|X||Y|$
 - b. $|X/Y|=|X|/|Y|$
 - c. $X/Y=|X|/|Y|e^{j\phi_x-\phi_y}$
 - d. $\operatorname{Re}[XY]=ac-bd$
 - e. $XX^*=|X|^2$
 - f. $(1/2)\operatorname{Re}[X^*Y]=(1/2)\operatorname{Re}[Y^*X]$
 - g. $|e^{jx}|=1$, x a real number.
 - h. $\cos x = \frac{1}{2}(e^{jx} + e^{-jx})$
 - i. $\sin x = \frac{1}{2j}(e^{jx} - e^{-jx})$
 - j. $e^{jX} = e^{-b}e^{ja}$
 - k. $|e^{jX}| = e^{-b}$
 - l. $e^{-jX} = e^be^{-ja}$
 - m. $|e^{-jX}| = e^b$

3. Given the quantity $Ce^{-jm\theta} + De^{jm\theta}$ is a real number, show that $D=C^*$, and that the real number is $2\operatorname{Re}[C]\cos(m\theta)+2\operatorname{Im}[C]\sin(m\theta)$.

4. What is the sound speed in
 - a. Air at 32°F, 70°F, 90°F.
 - b. H₂ at 32°F, 70°F, 90°F.
 - c. Distilled water at depths of 2m, 5m, 100m and temperatures of 45°F and 70°F.
 - d. Seawater at depths of 2m, 5m, 100m and temperatures of 45°F and 70°F. Assume that the salinity of the seawater is 35 parts per thousand, and that the gage pressure can be estimated from ρgz , where z is the depth.

5. A rightward traveling acoustic plane wave, propagating in air at a frequency of 5 kHz is specified by a $\hat{p}(x,t) = (-0.5 + 3j)e^{j(\omega t - kx)}$ Pa.
- Is this a transient or steady state acoustic wave?
 - Compute the acoustic pressure $p(x,t)$?
 - What is the frequency ω ?
 - What is the pressure amplitude?
 - What is the RMS acoustic pressure amplitude?
 - What is the SPL in dB re 20 μ Pa?
 - What is the wavelength λ ?
 - What is the value of the wavelength?
 - What is the value of the wavenumber?
 - What is the acoustic velocity $u_x(x,t)$?
 - What is the acoustic velocity vector \vec{u} ?
 - What is the acoustic velocity amplitude?
 - What is the acoustic displacement $\xi_x(x,t)$?
 - What is the acoustic displacement amplitude?
 - What is the total peak-to-peak acoustic particle movement during an acoustic cycle.
 - What is the intensity?
 - What acoustic power will be transported by the wave through an area of 10 cm^2 ?
 - If the plane wave traveled leftward, which quantities would change? Specify their new values.
6. An outward going spherical wave propagating in water at a frequency of 3 kHz is specified by $\hat{p}(r,t) = \frac{5 + 3j}{r} e^{j(\omega t - kr)}$ Pa. Using a distance of $r = 3.5$ m, repeat 4a-q for this acoustic wave.
7. For the acoustic waves given in problems 4 and 5, compute the velocity potentials $\hat{\phi}(x,t)$ and $\hat{\phi}(r,t)$.