ME413/MW513/ME504ST/ME523/EE579 HW#2

Propagation of Acoustic Waves in Rectangular, Cylindrical and Spherical Coordinates

1. Problems 6.4.1, 6.4.2, 6.4.5 (except 6.4.5c), 6.47C from the class text.

2. For a function $f(x)$ with period ℓ over the domain $0<x<\ell$

$$f(x) = \begin{Bmatrix} \frac{2k}{\ell}x & , & 0 < x < \frac{\ell}{2} \\ \frac{2k}{\ell}(\ell - x) & , & \frac{\ell}{2} < x < \ell \end{Bmatrix}$$

   a. Determine the Fourier series coefficients C_m and spatial frequencies $m\omega$ for a series of the form $f(x) = \sum_{m=0}^{\infty}\left[C_m e^{-jm\omega x} + C_m^* e^{jm\omega x}\right]$. Print the numerical values of the first six coefficients C_m, m=0,1,..5.
   b. Plot several cycles of the original function, and the Fourier series with 3, 10, and 50 terms. Use $\ell=2$ and k=1 for your plot.

3. For the function $f(x,y)$ that is periodic in x and y with periods ℓ_x and ℓ_y respectively

$$f(x,y) = \begin{Bmatrix} 1 & , & \frac{\ell_x}{4} < x < \frac{3\ell_x}{4} \text{ and } \frac{\ell_y}{4} < y < \frac{3\ell_y}{4} \\ & 0 & , \quad \text{elsewhere} \end{Bmatrix}$$

   a. Generate a double Fourier series from the product of two Fourier series.
   b. Generate a surface plot of the double Fourier series using 20 terms from each of the component series. Use $\ell_x = \ell_y = 1$ for your plot.

4. Consider the radiation of a patch on an infinite cylinder of diameter $2a$ into an acoustic medium. Specifically, the boundary conditions for this problem are

$$\hat{u}_r \,|_{r=a}\; e^{j\omega t} = \begin{Bmatrix} Ue^{j\omega t} & , & -\frac{\pi}{4} < \theta < \frac{\pi}{4} \\ 0 & , & \text{elsewhere} \end{Bmatrix}$$

   a. Generate an expression for the complex acoustic pressure amplitude.
   b. Assuming the medium is air, the diameter is $2a$= 20 cm, U=1 mm/sec, and a frequency of 1000 Hz, plot the acoustic pressure amplitude out to a distance of 3 wavelengths.

5. **ME513/ME504ST/ME523/EE579 Students**. Consider radiation from patches that are bounded and periodic in the z coordinate with period ℓ_z, and

are also bounded in the angular coordinate θ. The domain of the period in the z coordinate is $-\ell_z/2 < z < \ell_z/2$. In this circumstance, the boundary conditions are

$$\hat{u}_r\,|_{r=a}\, e^{j\omega t} = \begin{Bmatrix} Ue^{j\omega t} & , & -\frac{\pi}{4} < \theta < \frac{\pi}{4} \text{ and } -\frac{\ell_z}{8} < z < \frac{\ell_z}{8} \\ & 0 & , \text{ elsewhere} \end{Bmatrix}$$

a. Generate an expression for the complex acoustic pressure amplitude.

b. Assuming the medium is air, the diameter 2*a*=10 cm, ℓ_z = 40 cm, *U*=1 mm/sec, and the frequency is 400 Hz, plot contour slices of the acoustic pressure amplitude out to 3 wavelengths at *z*=0, *z*=$\ell_z/8$, and *z*=$\ell_z/2$.

6. Consider the motion of a transversely oscillating sphere. The sphere, of radius *a*, oscillates back and forth along the *z* axis at velocity $Ue^{j\omega t}$.

   a. Use a diagram to how that a boundary condition of $\hat{u}_r(a,\theta,\phi)e^{j\omega t} = U\cos(\theta)e^{j\omega t}$ is an appropriate description of the transversely oscillating sphere.

   b. Generate an expression for the acoustic pressure amplitude caused by a transversely oscillating sphere.

7. Consider the function *f*(θ) periodic over the interval 0<*θ*<π

$$f(\theta) = \begin{Bmatrix} 1 & , & 0 < \theta < \frac{\pi}{8} \\ 0 & , & \text{elsewhere} \end{Bmatrix}$$

   a. Derive the first four coefficients A_m of a Legendre series $f(\theta) = \sum_{m=0}^{\infty} A_m P_m(\cos\theta)$ algebraically.

   b. Use software to compute A_m, *m* ranging to an arbitrary number *M*. Plot the Legendre series for *M*=10, 25, 100, and the original function *f*(θ).

8. **ME513/ME504ST/ME523/EE579 Students**. Consider the radiation from a patch on a sphere of radius *a*. The boundary condition for this problem is

$$\hat{u}_r\,|_{r=a}\, e^{j\omega t} = \begin{Bmatrix} Ue^{j\omega t} & , & 0 < \theta < \frac{\pi}{8} \\ 0 & , & \text{elsewhere} \end{Bmatrix}$$

   a. Generate an expression for the complex acoustic pressure amplitude.

b. Assuming the medium is air, the diameter $2a$=10 cm, and the frequency is 400 Hz, plot a contour slice and surface in the xz plane of the acoustic pressure amplitude out to 3 wavelengths.