ME413/MW513/ME504ST/ME523/EE579 HW#4

Waveguides and Cavities

1. Consider an air-filled, semi-infinite rectangular waveguide. The source is located at z=0, and the dimensions in the *x* and *y* directions are L_x=50 cm and L_y=75 cm respectively. The source is defined by a velocity amplitude $\hat{u}_z(x,y,0)$ in the *z* direction of

$$\hat{u}_z(x,y,0)e^{j\omega t} = \begin{Bmatrix} U_o e^{j\omega t} \;, 0 < x < \dfrac{L_x}{2}, 0 < y < \dfrac{L_y}{2}, \\ 0, \text{ elsewhere} \end{Bmatrix}$$

where $\omega = 2\pi f$= 4398 rad/sec and U_o=1 mm/sec.

   a. Determine the cutoff frequency.
   b. How many modes propagate?
   c. What are the propagation directions relative to the z axis for the propagating modes?
   d. What are the phase velocities for the propagating modes?
   e. Determine the absorption coefficient α, assuming that the temperature is 60^{o}F and the relative humidity is 50%.
   f. Plot the pressure amplitude in the *yz* plane at $x=L_x/2$ to $z=3\lambda$. Include mode indexes up through l=10 and m=10.
   g. Plot the amplitude of the *z* component of the acoustic velocity in *yz* plane at $x=L_x/2$ to $z=3\lambda$. Include mode indexes up through l=10 and m=10.

2. Consider a rectangular room. The origin of an *xyz* coordinate system is located in one corner of the room. The dimensions of the room in the *x*,*y*, and *z* directions respectively are L_x=20ft, L_y=10ft, and L_z=8ft. The room is filled with air.

   a. Compute the first 10 natural frequencies of the room.
   b. How many natural frequencies exist between in the frequency range 1-2kHz?

3. **ME513/ME504ST/ME523/EE579 Students**. In the design of cabinets for loudspeakers, it is required to know the "spring-iness" of the air in the cabinet, as it is observed by the loudspeaker diaphragm. Suppose a loudspeaker diaphragm is modeled by a rigid circular disk of radius a=5 cm. This disk is mounted to one wall of a cabinet, that takes the form of a rectangular parallelepiped. The origin of an *xyz* coordinate system is located in one corner of the cabinet, and the dimensions of the box in the *x*, *y*, and *z* directions are L_x=20 cm, L_y=50 cm, and L_z=20 cm respectively. The center of the rigid disk is located at (*x*,*y*,*z*) = (10cm, 10cm,0), and vibrates with a velocity amplitude

in the z direction of U_o =10 mm/sec. To estimate absorption in the cabinet, use a temperature of 60^oF and a relative humidity of 50%.

a. At frequencies of f=100, 500, 2000, and 5000 Hz, how many modes are propagating?
b. Plot the acoustic pressure amplitude in the cabinet versus yz at $x=L_x/2$ at a frequency of 2000 Hz.
c. Plot the magnitude of the force amplitude $\hat{F} = \int_S \hat{p} dS$, where S is the disk surface, versus frequency over the range 100-5000 Hz. Include up through $l=m=20$ modes in your calculation.
d. Plot the magnitude and phase of the impedance $z=F/U_o$ versus frequency over the range 100-5000 Hz. Compare to the approximate impedance $z = \frac{1}{j\omega}\frac{\rho_o c^2 S^2}{V}$, where $S=\pi a^2$ is the area of the rigid disk, and $V=L_xL_yL_z$ is the volume of the cabinet.

4. Consider a water-filled, seminfinite cylindrical pipe 10 cm in diameter. An cylindrically symmetric acoustic source is located at z=0, and take the form

$$\hat{u}_z(r,0)e^{j\omega t} = \begin{Bmatrix} U_o e^{j\omega t}\ , 0.02 < r < 0.03, \\ 0, \text{elsewhere} \end{Bmatrix}$$

where U_o= -0.5 mm/sec, and $\omega/2\pi=f$=25 kHz.. You may neglect absorption.

a. What is the cutoff frequency for the pipe?
b. How many modes propagate in the pipe?
c. Plot the pressure amplitude in the rz plane out to $z=3\lambda$. Include the propagating modes, and 5 evanescent modes in your acoustic pressure..
d. Plot the amplitude of the z component of the acoustic velocity in rz plane out to $z=3\lambda$. Again, include the propagating modes, and 5 evanescent modes in your acoustic pressure

5. Consider an air-filled cylindrical cavity of diameter 10 cm, and length 75 cm. What are the first 10 resonance frequencies?