University of Idaho Continuing

$$\rho_0 (j\omega) \hat{U} \cdot e^{j(\omega t - kr)} = -\frac{\partial}{\partial r}\left[\frac{\hat{A}}{r} e^{j(\omega t - kr)}\right]$$

$$= -\left[(-jk)\frac{\hat{A}}{r} e^{j(\omega t - kr)} - \frac{\hat{A}}{r^2} e^{j(\omega t - kr)}\right]$$

So, we determine the complex velocity amplitude $\hat{U}$ to be:

$$\hat{U} = \frac{1}{\rho_0 c}\frac{\hat{A}}{r}\left[1 - \frac{j}{kr}\right]$$

University of Idaho **So, the acoustic velocity is**

$$\hat{u}(r,t) = \frac{1}{\rho_0 c}\frac{\hat{A}}{r}\left[1 - \frac{j}{kr}\right]e^{j(\omega t - kr)} = \hat{U} e^{j(\omega t - kr)}$$

Recall: $\hat{p}(r,t) = \frac{\hat{A}}{r} e^{j(\omega t - kr)}$

Note: as $kr \to \infty$ $\hat{u}(r,t) = \frac{1}{\rho_0 c}\frac{\hat{A}}{r} e^{j(\omega t - kr)}$

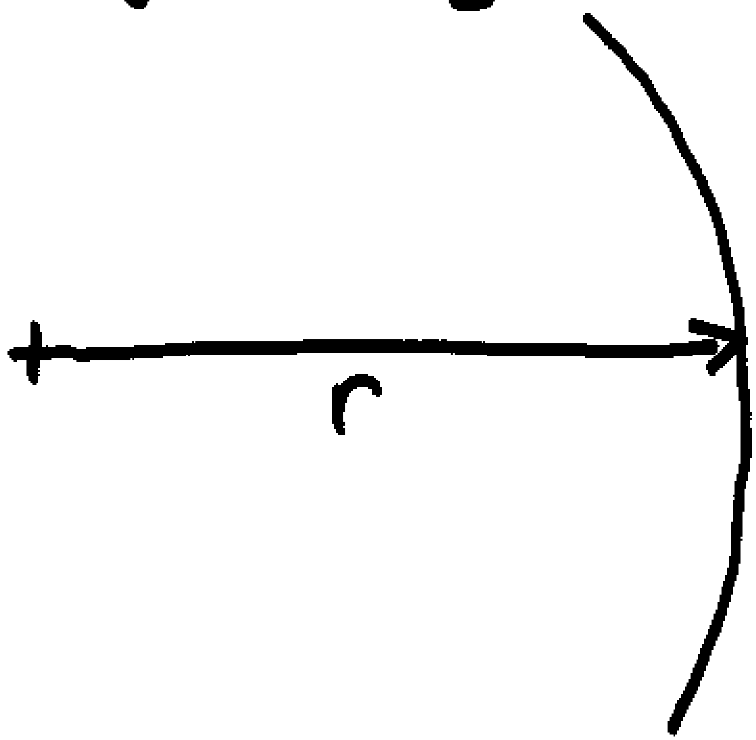
In this limit; the specific acoustic impedance is:

little p

$$\hat{z} = \frac{\hat{p}}{\hat{u}} \Rightarrow \lim_{kr \to \infty} \frac{\hat{p}}{\hat{u}} = \frac{\hat{A}/r}{\frac{\hat{A}}{r}\frac{1}{\rho_0 c}} = \rho_0 c$$

University of Idaho **What is kr??**

$$kr = \frac{\omega}{c} r = \frac{2\pi f}{c} r = 2\pi\left(\frac{r}{\lambda}\right)$$



If $r \gg \lambda$ then the spherical wave is physically like a plane wave.

Expand $kr \gg 1 \Rightarrow \cancel{r \gg 1.59}\lambda \quad r \gg 0.159\lambda$

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Lets proceed to compute the acoustic intensity

$$(AB)^* = A^* B^*$$

$$\vec{I} = \frac{1}{2} \mathrm{Re}\left[\hat{P}\,\hat{U}^*\right]\hat{r}$$

$$= \frac{1}{2} \mathrm{Re}\left[\frac{\hat{A}}{r}\left[\frac{\hat{A}}{r}\frac{1}{\rho_0 c}\left(1 - \frac{j}{kr}\right)\right]^*\right]\hat{r}$$

$$= \frac{1}{2} \mathrm{Re}\left[\frac{|\hat{A}|^2}{r^2}\frac{1}{\rho_0 c}\left(1 + \frac{j}{kr}\right)\right]\hat{r}$$

$$= \frac{1}{2\rho_0 c}\frac{|\hat{A}|^2}{r^2} = \frac{1}{2\rho_0 c}\left(\frac{|\hat{A}|}{r}\right)^2\hat{r}$$

pressure amplitude at r

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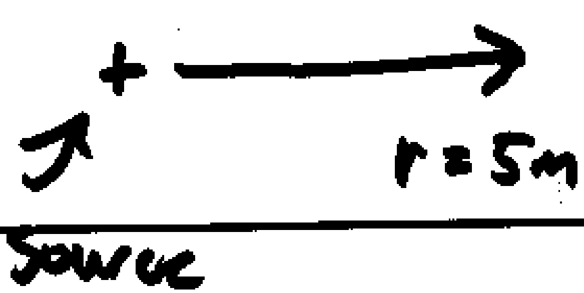
Lets compute the acoustic power output "at a distance" r:

$$\Pi = \int_A \vec{I}\cdot\vec{n}\,dA = I * \text{Area} = \frac{1}{2\rho_0 c}\left(\frac{|\hat{A}|}{r}\right)^2 4\pi r^2$$

$$= \frac{2\pi|\hat{A}|^2}{\rho_0 c}$$

Consider the example



- We measure a pressure amplitude of 1 Pa
- How do we calculate Π??

 $1\,Pa \Rightarrow \frac{|\hat{A}|}{r} = 1\ Pa$

Suppose we wish to calculate the ~~dist~~ acoustic <u>displacement</u> amplitudes of the particles in a medium.

For a harmonic problem, we know how to calculate <u>acoustic velocity</u>

$$\hat{u}(x,t) = \hat{U} e^{j(\omega t - kx)} \quad ; \quad \hat{U} = \text{a known \#}$$

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The acoustic displacement is

$$\hat{\xi}(x,t) = \underset{\uparrow\,\xi}{\hat{\xi}}\, e^{j(\omega t - kx)} \quad ; \quad \hat{\xi} = \text{acoustic displacement amplitude.}$$

The relationship between acoustic particle velocity and displacement is

$$u = \frac{d\xi}{dt} \Rightarrow \hat{U} e^{j(\omega t - kx)} = (j\omega)\,\hat{\xi}\, e^{j(\omega t - kx)}$$

$$\hat{\xi} = \hat{U}/j\omega \quad \therefore \quad |\hat{\xi}| = \frac{|\hat{U}|}{\omega}$$

University of Idaho **Condensation**

$$s = \cancel{\frac{\rho' - \rho_0}{\rho_0}} = \frac{\rho'}{\rho_0} - 1 \Rightarrow \hat{s} = \frac{1}{\rho_0}\hat{\rho}' - 1$$

$$= \frac{\rho'}{\rho_0} \Rightarrow \hat{s} = \frac{\hat{\rho}'}{\rho_0}$$

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Sound measurement

Apparatus :