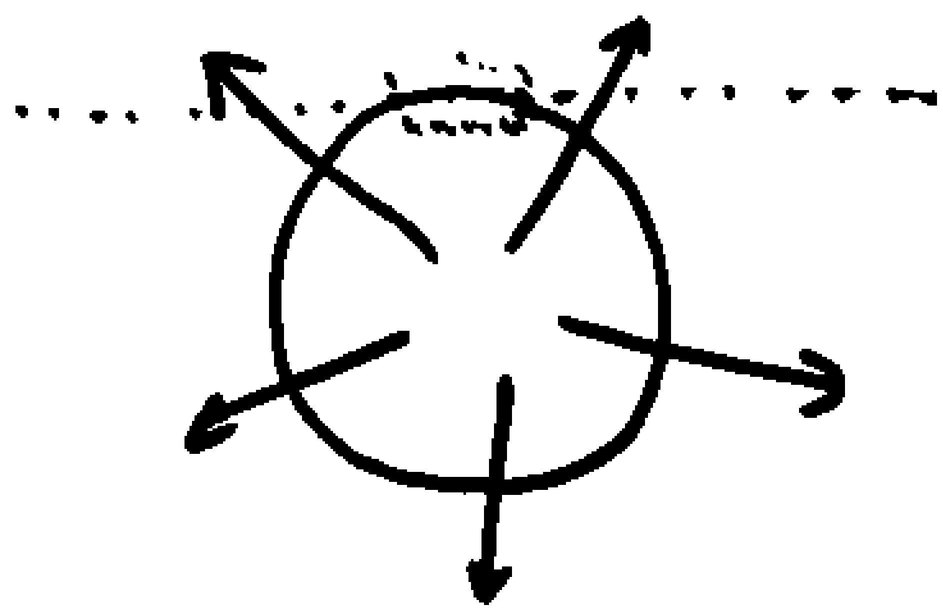
University of Idaho **Hw #2 Due Wed Feb 6** ~

Counter-point is a cylindrical wave on the surface of the water, looking down



University of Idaho

The mathematical representation of a plane wave propagating parallel to the x axis is

$$p(x,t) = f(\underline{x-ct}) \equiv$$ D'Alembert's solution to the wave equation

contains the wave shape

⇒ Transient or steady state

Propagates the wave "to the right" at speed c.

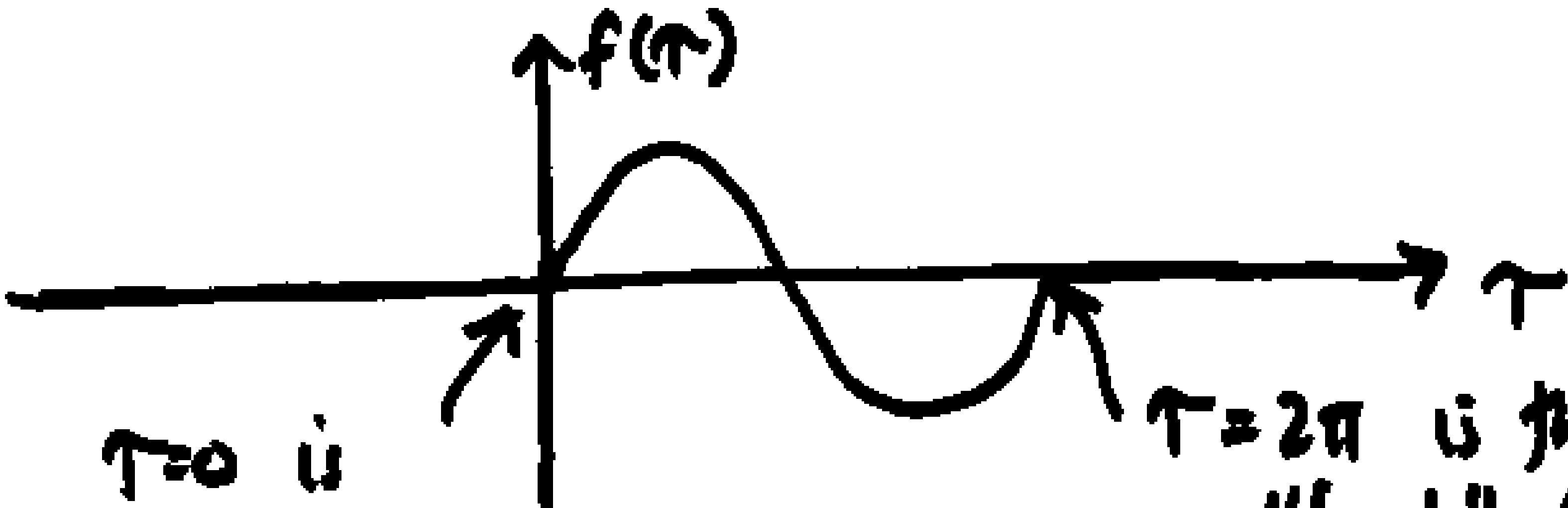
University of Idaho

Propagation Part: Define

a wave shape like

$$f(\tau) = \begin{cases} 0 & \tau > 2\pi \\ \sin(\tau) & 0 \le \tau \le 2\pi \\ 0 & \tau < 0 \end{cases}$$

$\tau = x - ct$

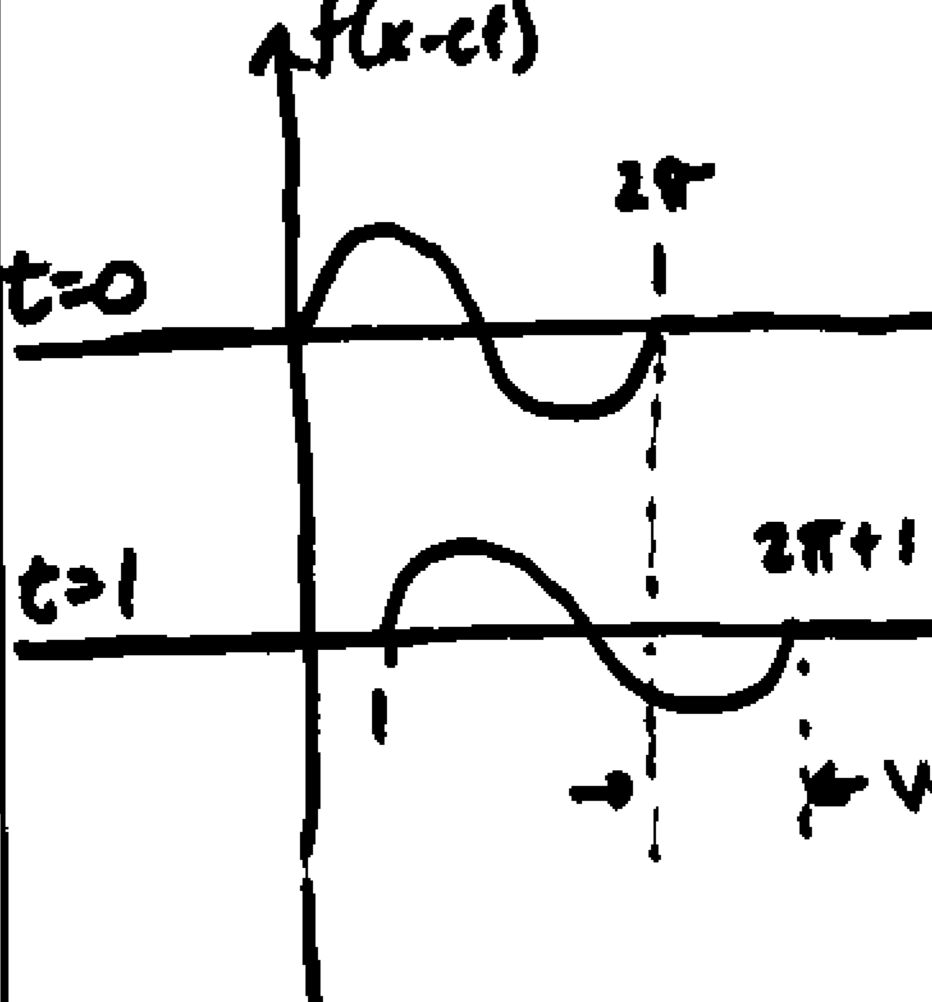


$\tau = 0$ is the "back" of the wave shape

$\tau = 2\pi$ is the "front" of the shape

University of Idaho

Let $c = 1\ m/s$, consider the location of the wave at various times.



$\tau = 2\pi = x - ct \Rightarrow x = 2\pi$

$\tau = 0 = x - ct \Rightarrow x = 0$

$\tau = 2\pi = x - c \cdot 1 \Rightarrow x = 2\pi + 1$

$\tau = 0 = x - c1 \Rightarrow x = 1$

velocity $= \dfrac{\Delta x}{\Delta t} = \dfrac{1}{1} = 1\ m/s$

University of Idaho How does it satisfy the wave equation?? Wave equation was

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

If $p = p(x, y, z, t)$, (x, y, z) are p rectangular coordinates. Then, the <u>Laplacian</u> operator

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}$$

University of Idaho

§ the wave equation is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

Our solution is $p = f(\tau)$, $\tau = x - ct$

For a plane wave, we need only satisfy the "1-D" wave equation:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

University of Idaho

$$\frac{\partial p}{\partial x}=\frac{\partial f}{\partial \tau}\frac{\partial \tau}{\partial x} \quad ; \quad \frac{\partial \tau}{\partial x}=1$$

$$\frac{\partial^2 p}{\partial x^2}=\frac{\partial}{\partial x}\left[\frac{\partial f}{\partial \tau}\right]=\frac{\partial^2 f}{\partial \tau^2}\frac{\partial \tau}{\partial x}=\frac{\partial^2 f}{\partial \tau^2}$$

By chain rule

$$\cancel{\frac{\partial p}{\partial t}=\frac{\partial f}{\partial \tau}\frac{\partial \tau}{\partial t}} \quad \frac{\partial p}{\partial t}=\frac{\partial f}{\partial \tau}\frac{\partial \tau}{\partial t} \Rightarrow \frac{\partial \tau}{\partial t}=-c$$

$$=\frac{\partial f}{\partial \tau}(-c)$$

$$\frac{\partial^2 p}{\partial t^2}=-c\frac{\partial}{\partial t}\left[\frac{\partial f}{\partial \tau}\right]=-c\frac{\partial^2 f}{\partial \tau^2}\frac{\partial \tau}{\partial t}=c^2\frac{\partial^2 f}{\partial \tau^2}$$

University of Idaho Put into the wave equation

$$\frac{\partial^2 f}{\partial \tau^2} - \frac{1}{c^2} c^2 \frac{\partial^2 f}{\partial \tau^2} \stackrel{?}{=} 0$$

$p = f(x - ct)$ satisfies the wave equation for <u>any</u> f !!

Some nominal sound speeds are

1) $c \approx 343 \, m/s \approx 1000 \, ft/sec$ (air)

2) $c \approx 1500 \, m/s$ (water)

University of Idaho

A more general form of D'Alembert's solution is:

$$p(x,t) = f(x-ct) + g(x+ct)$$

We can write solutions for waves propagating in the y or z dirs as

$$p(y,t) = f(y-ct) + g(y+ct)$$

$$p(z,t) = f(z-ct) + g(z+ct)$$