ME 413/513 HW #5 Solutions

6.2.1

water: $\rho = 1000$ $c = 1500$ | air: $\rho = 1.21$ $c = 343$

$P_c = 50$ Pa

$f = 1$ kHz

a) $\hat{T} = \dfrac{2r_a}{r_a + r_w}$ $\quad r_a = 1.21(343) = 415$

$r_w = 1000(1500) = 1.5\times10^6$

$$\hat{T} = \frac{2(415)}{415 + 1.5\times10^6} = 553.2\times10^{-6}$$

$$P_e \text{ transmitted} = (553.2\times10^{-6})(50) = 27.7 \text{ mPa}$$

b) $I_w = \dfrac{1}{2}\dfrac{(\sqrt{2}\,50)^2}{1000\cdot1500} = 1.67 \text{ mW/m}^2$ $\qquad I_a = \dfrac{1}{2}\dfrac{(\sqrt{2}\cdot27.7\times10^{-3})^2}{1.21(343)} = 1.85\,\mu\text{W}$

c) $$\text{dB reduction} = 10\log\left[\frac{I_a}{I_w}\right] = 10\log\left[\frac{1.85\times10^{-6}}{1.67\times10^{-3}}\right] = -29.6 \text{ dB}$$

d) water: $\rho = 1000,\ c = 1500$ | ice: $\rho = 920,\ 3200$

$r_w = 1.5\times10^6$ $\quad r_{ice} = 2.94\times10^6$

$$\hat{T} = \frac{2(2.94\times10^6)}{2.94\times10^6 + 1.5\times10^6} = 1.32$$

$P_e = 1.32(50) = 66.2$ Pa $\qquad I_w = 1.67 \text{ mV/m}^2$ $\qquad I_{ice} = \dfrac{(\sqrt{2}\cdot66.2)^2}{2(920)(3200)} = 1.49 \text{ mV}$

$$\text{dB reduction} = 10\log\left[\frac{I_{ice}}{I_w}\right] = 10\log\left[\frac{1.49}{1.67}\right] = -0.495 \text{ dB}$$

6.2.2

$P_r \uparrow$ $\quad \downarrow P_i$ $\quad$ sea water

sediment

P_r is 20 dB less than P_i $\therefore$ $P_r = 0.1 P_i$

$|\hat{R}| = 0.1$ $\qquad K = \dfrac{r_{sed} - r_{sea}}{r_{sea} + r_{sed}}$

$r_{sea} = 1026(1500) = 1.539\times10^6$ Rayls

$$\frac{r_{sed} - r_{sea}}{r_{sea} + r_{sed}} = 0.1 \Rightarrow 10r_{sed} - 10r_{sea} = r_{sea} + r_{sed}\ ; \quad 9r_{sed} = 11r_{sea}$$

$$r_{sed} = \frac{11}{9}(1.539\times10^6) = 1.881\times10^6 \text{ Rayl}$$

Or

$$\frac{r_{sea}-r_{sed}}{r_{sea}+r_{sed}} = -0.1 \qquad -10r_{sea}+10r_{sed} = r_{sea}+r_{sed} \; ; \; 9r_{sed} = 11r_{sea}$$

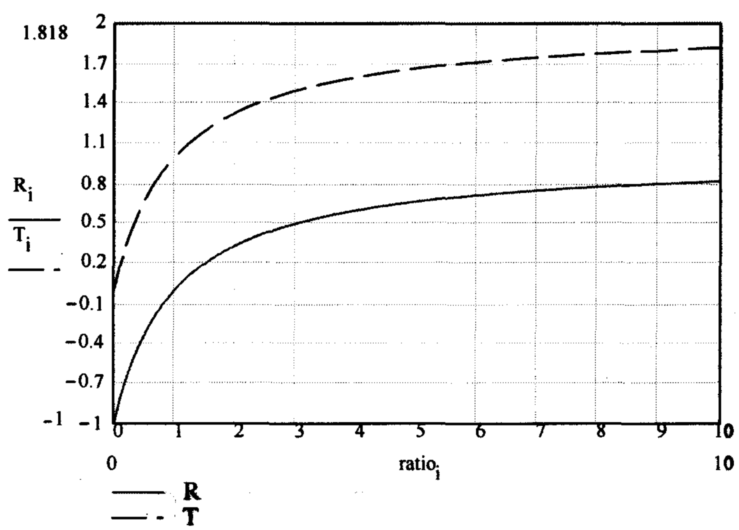
$$r_{sed} = \frac{11}{9}r_{sea} = \frac{11}{9}(1.539\times10^6) = 1.88\times10^6 \text{ Rayl}$$

6.26C

$$\hat{R} = \frac{r_2/r_1 - 1}{r_2/r_1 + 1} \; ; \quad \hat{T} = \frac{2\, r_2/r_1}{r_2/r_1 + 1}$$

- R, T have reached 80% of their limiting values by $r_2/r_1 = 10$
- $r_2/r_1 > 1$; $R > 0$ and $r_2/r_1 < 1$, $R < 0$

$i := 0, 1 .. 100 \qquad ratio_i := \frac{i}{10} \qquad R_i := \frac{ratio_i - 1}{ratio_i + 1} \qquad T_i := \frac{2\cdot ratio_i}{ratio_i + 1}$



6.3.2

water	Plastic	Steel
$\rho = 1000$	$\rho_p = 1500$	$\rho = 7700$
$c = 1500$	c_p ?	$c = 6100$

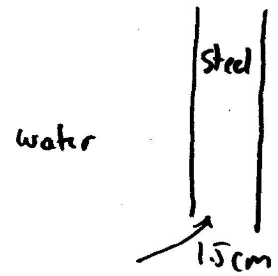
want to get energy from water to steel ($r_1 \neq r_3$) ∴ use $\lambda/4$ layer of plastic

$f = 20$ kHz $\qquad \rho_p c_p = \sqrt{(1000)(1500)(7700)(6100)} = 8.39\times10^6$ Rayl $\qquad c_p = \frac{8.39\times10^6}{1500} = 5{,}596$ m/s

$$L = \frac{1}{4}\frac{c_p}{f} = \frac{1}{4}\frac{c_p}{20\times10^3} = \frac{5{,}596}{4(20\times10^3)} = 6.99 \text{ cm}$$

3/

6.3.3



$f = 2\ \text{kHz}$

water

$\rho = 1000,\ c = 1500$

$\rho = 7700 \quad c = 6100$

$$T_I = \frac{I_t}{I_i} = \frac{\frac{1}{2}\frac{P_t^2}{\rho_w c_w}}{\frac{1}{2}\frac{P_i^2}{\rho_w c_w}}$$

$$T_I = \frac{P_t^2}{P_i^2}$$

$$\log T_I = 2\log\left[\frac{P_t}{P_i}\right] \ ; \ 10\log T_I = 20\log\left[\frac{P_t}{P_i}\right]$$

So;

$$T_I = \frac{1}{1+\frac{1}{4}\left(\frac{r_2}{r_1}-\frac{r_1}{r_2}\right)^2\sin^2 k_2 L}$$

$r_1 = r_3 = 1.5\times10^6 \qquad r_2 = 7700(6100) = 49.97\times10^6$

$$= \frac{1}{1+\frac{1}{4}\left(\frac{49.97}{1.5}-\frac{1.5}{49.97}\right)^2\sin^2\left[\frac{2000(2\pi)}{6100}\cdot 0.015\right]} = 0.79$$

Then $10\log[T_I] = -1.02$ dB

If $T_I = 0.79$ then $R_I = 0.21 \quad (R_I + T_I = 1)$

Now, substitute sponge rubber $\rho = 500\ \text{kg/m}^3$ $c = 1000\ \text{m/s}$ for the steel

then $r_2 = (500)(1000) = 0.5\times10^6$ Rayl

$$T_I = \frac{1}{1+\frac{1}{4}\left(\frac{0.5}{1.5}-\frac{1.5}{0.5}\right)^2\sin^2\left[\frac{2000(2\pi)}{1000}\cdot 0.015\right]} = 0.94$$

and $10\log[T_I] = -0.263$ dB

and $R_I = 0.06$

6.3.4 water | ?? | steel

$\rho = 1000,\ c = 1500$ | ?? | $\rho = 7700$, $c = 6100$

$r_1 \neq r_3$, use $\lambda/4$ layer

a) $r_2 = \sqrt{r_1 r_3} = \sqrt{1000(1500)(7700)(6100)}$

$= 8.39\times10^6$ Rayl

b) $f = 20$ kHz ; $d = 0.01$ m ; $L = \frac{\lambda}{4} = \frac{c}{4f} \Rightarrow c = L\cdot(4f) = 0.01(4)(20\times10^3) = 800\ \text{m/s}$

$$\rho = \frac{8.39\times10^6\ \text{Rayl}}{800\ \text{m/s}} = 10{,}488\ \text{kg/m}^3$$