For a right word travelling wave;

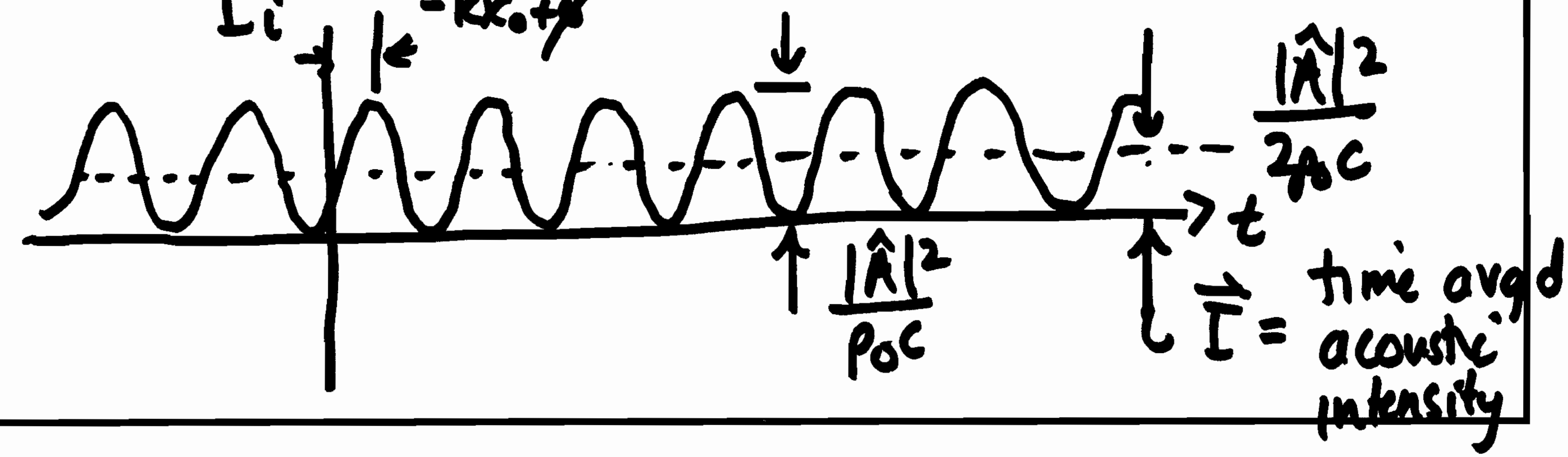
$\phi = \arg \hat{A}$

$$\hat{p}(x,t) = \hat{A} e^{j(\omega t - kx)} \qquad p(x,t) = |\hat{A}| \cos(\omega t - kx + \phi)$$

$$\hat{u}(x,t) = \frac{\hat{A}}{\rho_0 c} e^{j(\omega t - kx)} \quad ; \quad \vec{u}(x,t) = \frac{|\hat{A}|}{\rho_0 c} \cos(\omega t - kx + \phi)\,\hat{x}$$

At a given location, the ~~acoustic~~ instantaneous acoustic intensity $\vec{I}_i$ will look like

$$\cos^2 x = \tfrac{1}{2} + \tfrac{1}{2}\cos 2x$$



A general way to approach the computation of time-avg acoustic intensity is to use the "time-average-of-a-product" rule:

$$p(x,y,z,t) \Rightarrow \hat{P}(x,y,z)e^{j\omega t} \quad \Leftrightarrow \quad \overbrace{\hat{A}e^{-jkx}}^{\hat{P}(x,y,z)} e^{j\omega t}$$

$$u(x,y,z,t) \rightarrow \hat{U}(x,y,z)e^{j\omega t} \quad \Leftrightarrow \quad \underbrace{\frac{\hat{A}}{\rho_0 c}e^{-jkx}}_{\hat{U}(x,y,z)} e^{j\omega t}$$

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Then, in general:

$$\vec{\bar{I}} = \frac{1}{T}\int_0^T p(x,y,z,t)\,u(x,y,z,t)\,dt = \frac{1}{2}\mathrm{Re}\left[\hat{p}(x,y,z)\,\hat{u}^*(x,y,z)\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\hat{p}^*(x,y,z)\,\hat{u}(x,y,z)\right]$$

For the rightward travelling plane wave;

$$\vec{\bar{I}} = \frac{1}{2}\mathrm{Re}\left[\hat{A}e^{-jkx}\frac{\hat{A}^*}{\rho_0 c}e^{jkx}\right] = \frac{|\hat{A}|^2}{2\rho_0 c}\hat{x}$$

Numerical example: $\hat{A} = 1+0j$ Pa $\Rightarrow$ SPL $= 90.87$ dB re $20\,\mu$Pa

$$\vec{\bar{I}} = \frac{|1|^2}{2\cdot 1.21\cdot 343} = 1.21\ \mathrm{mW/m^2}$$

Compute acoustic intensity for a leftward-travelling plane wave;

$$\hat{p}(x,t) = \hat{A}e^{j(\omega t + kx)} = \hat{A}\,e^{jkx}\,e^{j\omega t}$$

$$\hat{u}(x,t) = -\frac{\hat{A}}{\rho_0 c}e^{j(\omega t + kx)} = -\frac{\hat{A}}{\rho_0 c}e^{jkx}\,e^{j\omega t} = (-1)\cdot\frac{\hat{A}}{\rho_0 c}(\;)$$

$$\vec{I} = \frac{1}{2}\mathrm{Re}\left[\hat{A}e^{jkx}(-1)\frac{\hat{A}^*}{\rho_0 c}e^{-jkx}\right] = -\frac{|\hat{A}|^2}{2\rho_0 c}\hat{x}$$

Regarding the text discussion of acoustic intensity:

We've specified acoustic pressure in terms of amplitude $|\hat{A}|$;

$$p(x,t) = |\hat{A}| \cos(\omega t - kx + \phi)$$

In terms of RMS pressure amplitude (effective pressure amplitude) p_e

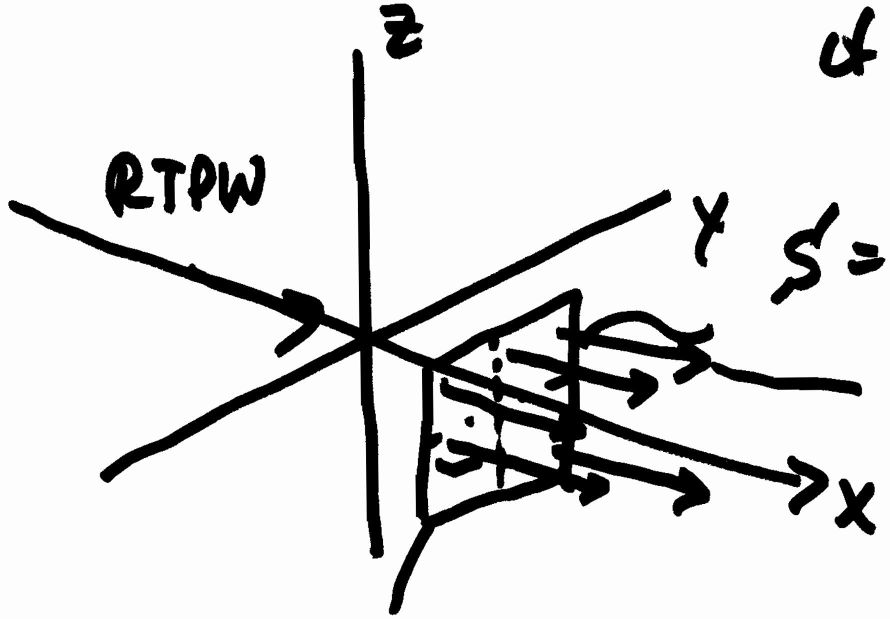
$$p_e = \frac{|\hat{A}|}{\sqrt{2}} \Rightarrow \bar{I} = \frac{|\hat{A}|^2}{2\rho_0 c} = \frac{p_e^2\, 2}{2\rho_0 c} = \frac{p_e^2}{\rho_0 c}$$

Acoustic Power:

Time averaged acoustic power $= \Pi = \int_{S} \vec{I} \cdot d\vec{S}$

dot product account for relative orientation of $\vec{I}$ and local surface area.

$\Rightarrow$ only want to count portion of $\vec{I}$ that is $\perp$ to S



$S = 1\,m^2$

$\vec{I}$ is $\perp$ to S everywhere

$\parallel$ to yz plane

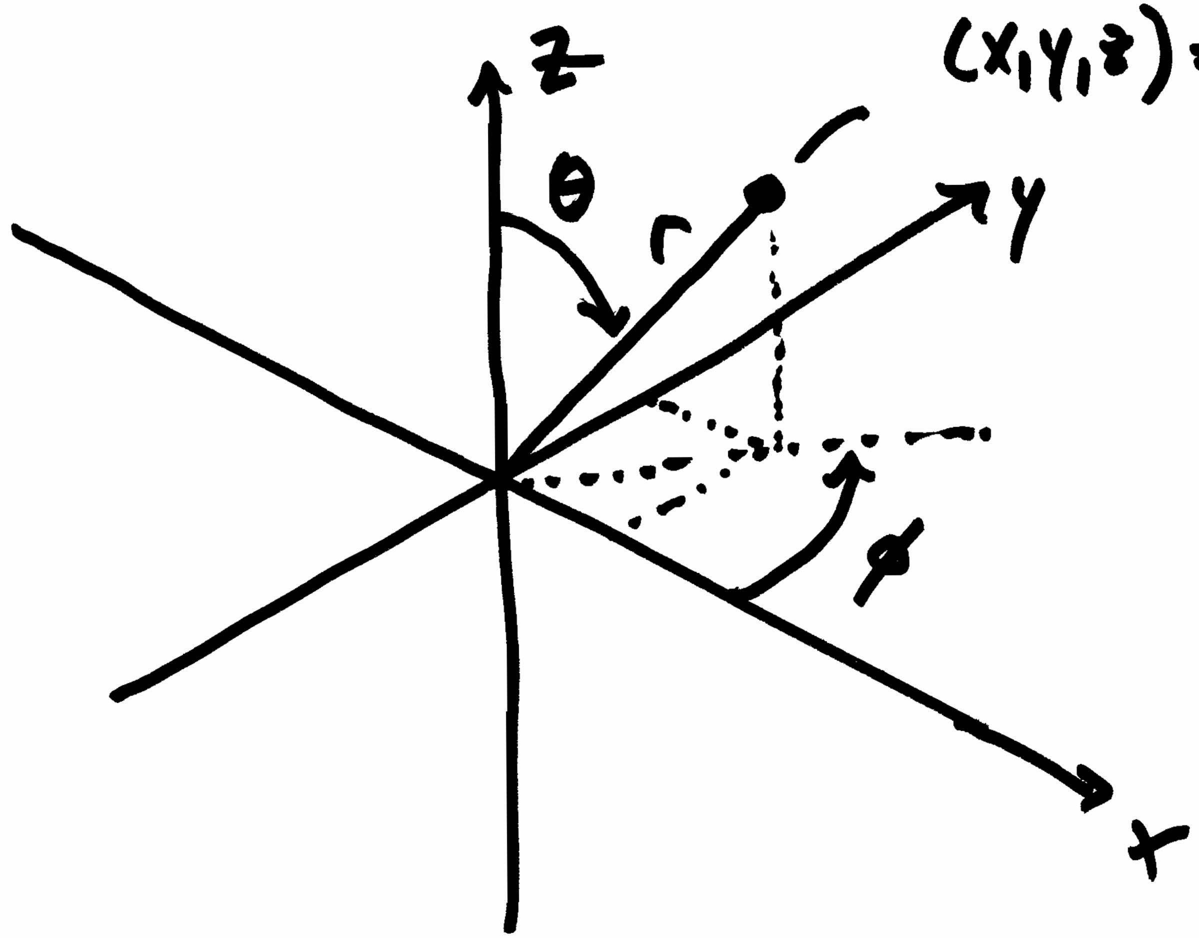
For a rightward travelling plane wave

$$\Pi = I S \sim \frac{W}{m^2} \cdot m^2 = W$$

From our previous example, with $S = 1 m^2$;

$$\Pi = 1.21 \frac{mW}{m^2} \cdot 1 m^2 = 1.21 mW$$

Spherical Waves



$(x, y, z) = (r, \theta, \phi)$

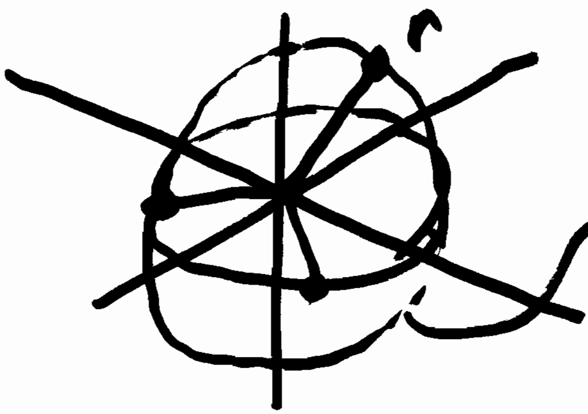
The wave equation is :

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \quad ; \quad p = p(r, \theta, \phi, t)$$

In spherical coords:, this expands to

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial p}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 p}{\partial\phi^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

Make the assumption that $p = p(r, t)$



"sphere of advancing phase"

Then the wave equation simplifies to

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

which is the same as

$$\frac{\partial^2 (rp)}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 (rp)}{\partial t^2} = 0$$

This is the wave equation in rp !

So, we know that $rp = f(x-ct) + g(x+ct)$