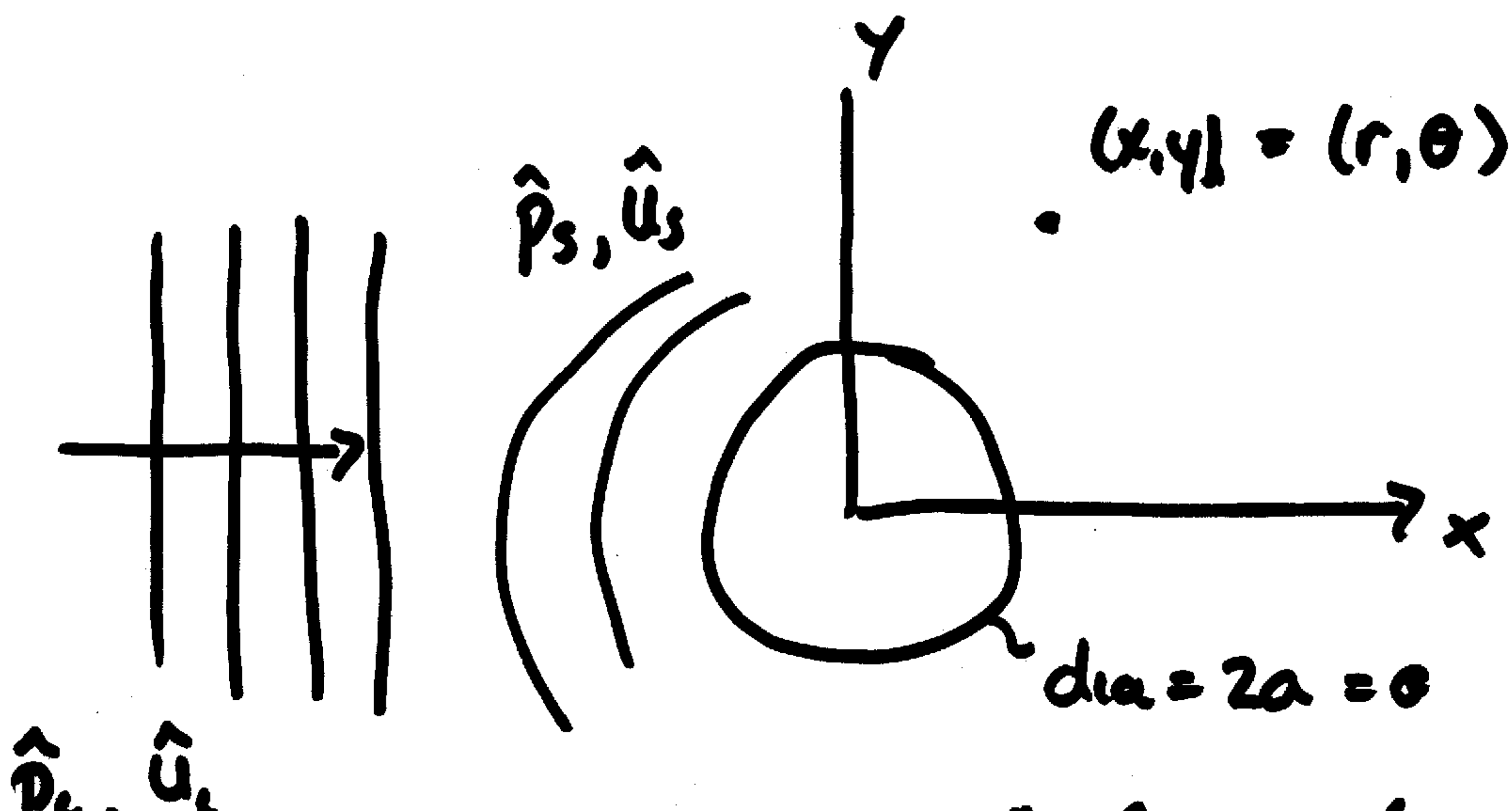
University of Idaho

Scattering From a Cylinder



Choose a proper scattered wave (radiated wave), such that

~~$\hat{u}_i(a,\theta) + \hat{u}_s(a,\theta) = 0$~~

University of Idaho

$$\hat{U}_{ri}(a,\theta) + \hat{U}_{rs}(a,\theta) = 0$$

The incident sa scatt wave is

$$\hat{p}_i = \hat{A}\, e^{j(\omega t - kx)}$$

~~$= \hat{A} J_0(kr) + \hat{A}\, 2 \sum$~~

$$= \hat{A} J_0(kr) + \hat{A}\, 2 \sum_{m=1}^{\infty} (-j)^m J_m(kr) \cos(m\theta)$$

~~$\hat{p}_s$~~

$$\hat{p}_s(r,\theta) = \sum_{m=0}^{\infty} A_m H_m^{(2)}(kr) \left[C_m e^{-jm\theta} + D_m e^{jm\theta} \right]$$

University of Idaho The incident and scattered radial acoustic velocities are $(\hat{A}) = 1$

$$\hat{u}_{Ri} = -\frac{\hat{A}1}{\rho_0 j\omega}\left\{-kJ_1(kr) + k\sum_{m=1}^{\infty}(-j)^m \cos(m\theta)\left[J_{m-1}(kr) - J_{m+1}(kr)\right]\right\}$$

$$\hat{u}_{Rs} = -\frac{1}{\rho_0 j\omega}\left\{-kH_1^{(2)}(kr)A_0[C_0 + D_0]\right.$$

$$\left. + \sum_{m=1}^{\infty}\frac{k}{2}\left[H_{m-1}^{(2)}(kr) - H_{m+1}^{(2)}(kr)\right]A_m[\;\cdot\;]\right\}$$

$[\;\cdot\;] = C_m e^{-jm\theta} + D_m e^{jm\theta}$

Need $C_m e^{-jm\theta} + D_m e^{jm\theta} = \cos m\theta \quad C_m = \frac{1}{2} + 0j,\ D_m = C_m^*$

University of Idaho

Then let $r=a$

$$\hat{u}_{rsi}(a,\theta) = -\frac{1}{\rho_0 j\omega}\left\{-kJ_1(ka) + k\sum_{m=1}^{\infty}(-j)^m\left[J_{m-1}(ka) - J_{m+1}(ka)\right]\cos(m\theta)\right\}$$

$$\hat{u}_{rs}(a,\theta) = -\frac{1}{\rho_0 j\omega}\left\{-kH_1^{(2)}(ka)A_0 + \sum_{m=1}^{\infty}\frac{k}{2}\left[H_{m-1}^{(2)}(ka) - H_{m+1}^{(2)}(ka)\right]A_m\cos(m\theta)\right\}$$

$$m=0 \Rightarrow \frac{kJ_1(ka)}{\rho_0 j\omega} + \frac{kH_1^{(2)}(ka)A_0}{\rho_0 j\omega} = 0$$

University of Idaho

$$m \geq 1 \Rightarrow -\frac{K(-j)^m[J_{m-1}(ka) - J_{m+1}(ka)]}{\rho_0 j\omega} - \frac{K[H_{m-1}^{(2)}(ka) - H_{m+1}^{(2)}(ka)]A_m}{2\rho_0 j\omega} = 0$$

$$\therefore \quad A_0 = - \frac{J_1(ka)}{H_1^{(2)}(ka)}$$

$$A_m = - \frac{2(-j)^m[J_{m-1}(ka) - J_{m+1}(ka)]}{H_{m-1}^{(2)}(ka) - H_{m+1}^{(2)}(ka)}$$

University of Idaho

$$\hat{p} = \hat{p}_i + \hat{p}_s \quad ; \quad \hat{p}_s = \sum_{m=0}^{\infty} A_m H_m^{(2)}(kr)\cos(m\theta)$$

$$\hat{p}_i = J_0(kr) + 2\sum_{m=1}^{\infty} (-j)^m J_m(kr)\cos(m\theta)$$

- Importance of $ka = \frac{2\pi}{\lambda}a = 2\pi\frac{a}{\lambda} = \pi\frac{2a}{\lambda}$
- Series becomes "m-impractical" to calculate when ka becomes large.
- ka very large, optic geometry