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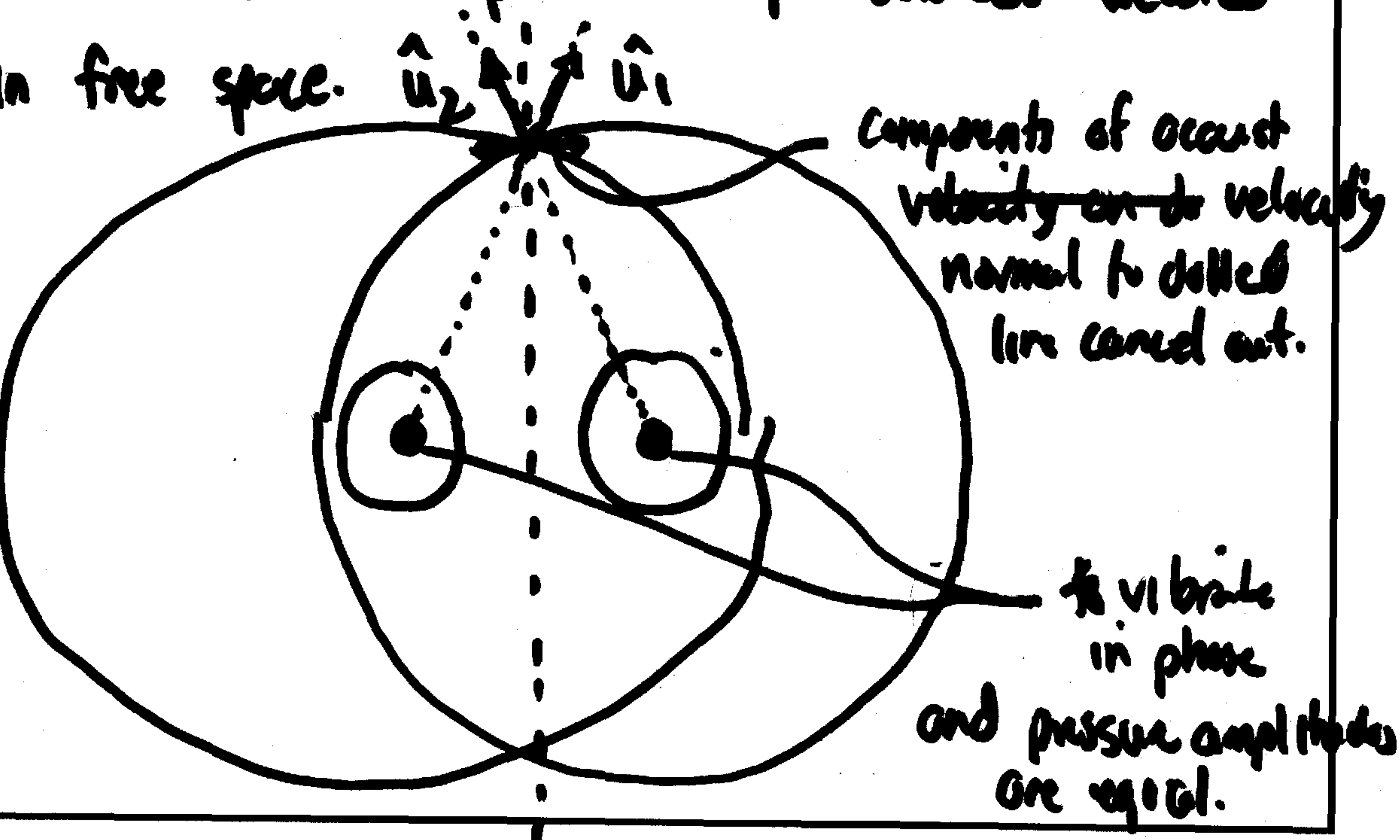
Method of Images

Consider two monopoles or simple sources located in free space.



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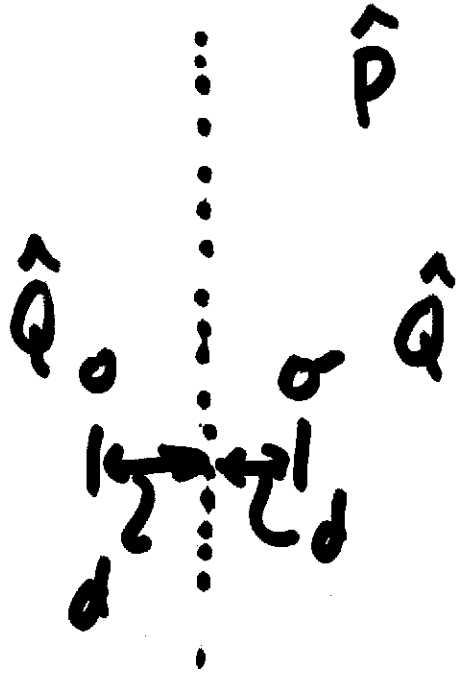
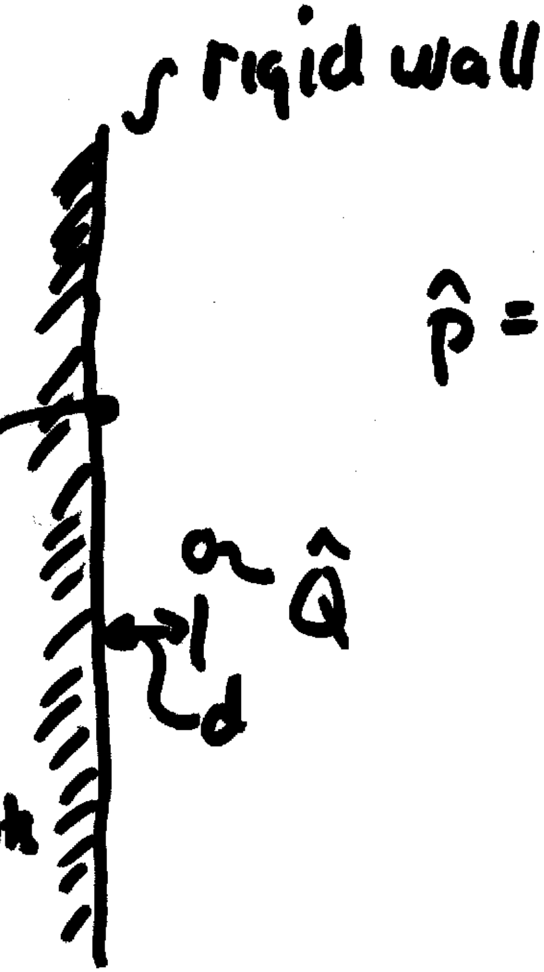


$\hat{p}$ given by sum of two sources, no-wall!!



$\hat{p} = ??$

BC is that particle movements normal to the wall are zero

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is not $d \ll \lambda$ ($kd \gg 1$), then the field $\hat{p}$ will have complex features of constructive and destructive interference

But, if $d \ll \lambda$ ($kd \ll 1$) then ~~the field~~ there is little destructive interference and

$$\hat{p} = 2j\rho c \frac{\hat{Q}k}{4\pi r} e^{j(\omega t - kr)}$$

Sound pressure caused by simple source near a rigid wall.

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$$I = \frac{1}{2}\rho_0 c\left[\frac{|\hat{Q}|^2}{\lambda r}\right]^2 \;;\; \Pi = \pi\rho_0 c\left[\frac{|\hat{Q}|^2}{\lambda}\right]$$

4x that in free space

2x in free space

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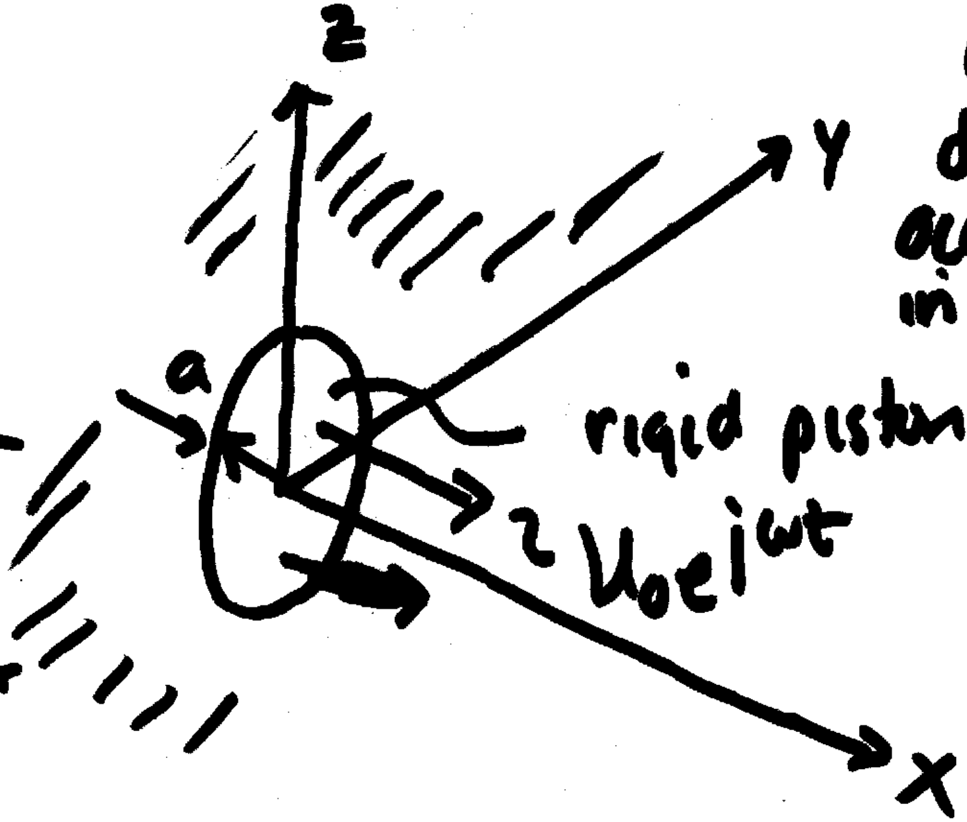
Plane Piston Problem

~~Mtd.~~ Model used to predict the performance of ~~acoustic~~ "real" acoustic receivers and transmitters.

we wish to determine the acoustic pressure in region $x>0$

rest of yz plane is a "baffle", infinite rigid planar wall



rigid piston, in yz plane

U_0 = velocity amplitude of rigid piston

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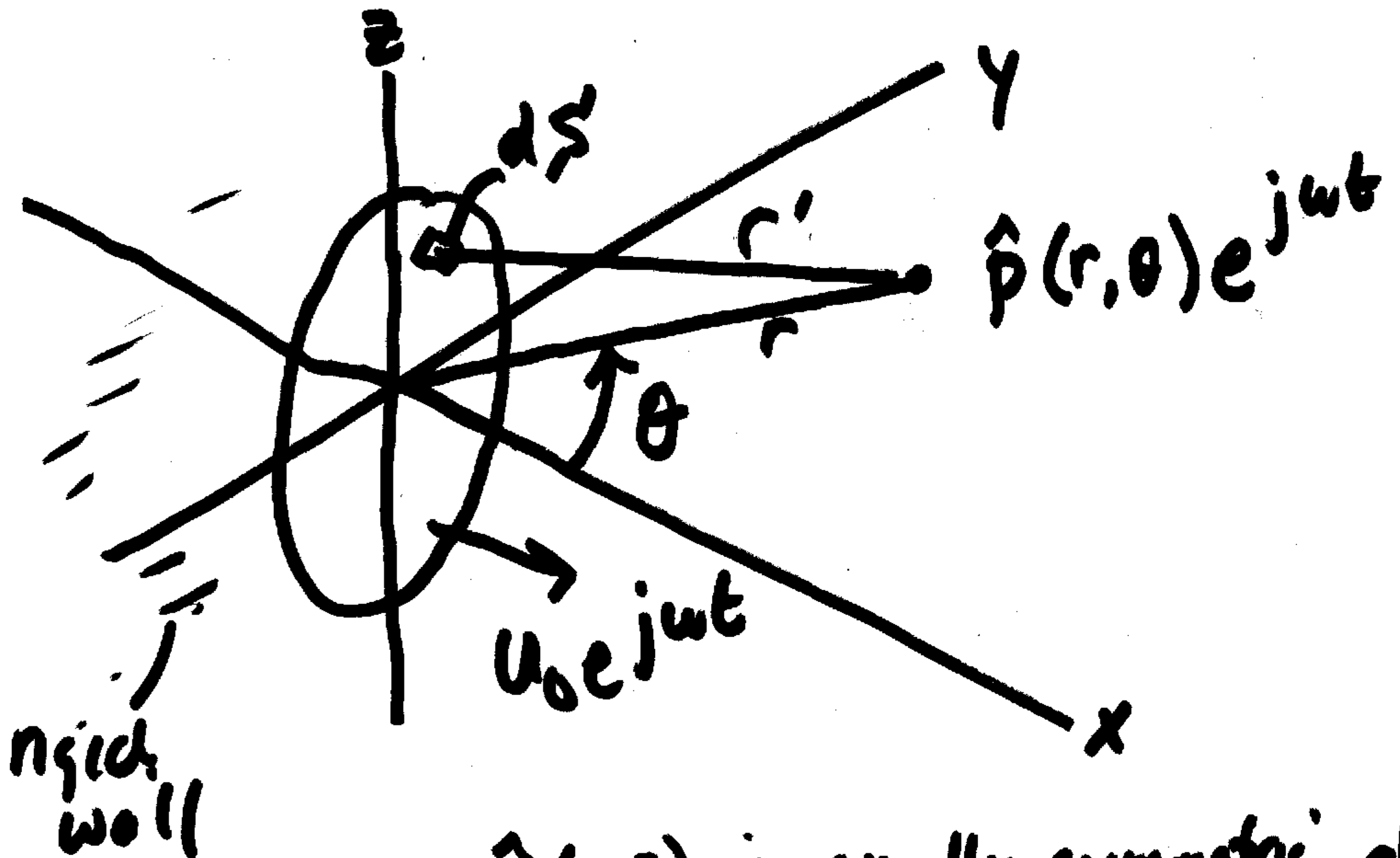

$\hat{p}(r,\theta)$ is axially symmetric about x axis

University of Idaho The contribution $d\hat{p}$ to $\hat{p}(r,\theta)$ caused by the vibration of dS' is:

$$d\hat{p} = 2\,j\rho_0 c\,\frac{(U_0\,dS')\,k}{4\pi r'}\,e^{j(\omega t-kr')}$$

Then, the total pressure $\hat{p}(r,\theta)e^{j\omega t}$ is

$$\underbrace{\hat{p}(r,\theta)}_{e^{j\omega t}} = \int_{S'} d\hat{p} = \int_{S'} 2j\rho_0 c\,\frac{U_0\,dS'\,k}{4\pi r'}\,e^{-jkr'}\,e^{j\omega t}$$

$$\hat{p}(r,\theta)e^{j\omega t} = j\rho_0 c\,\frac{U_0 k}{2\pi}\int_{S'}\frac{e^{-jkr'}}{r'}\,dS'\,e^{j\omega t}$$