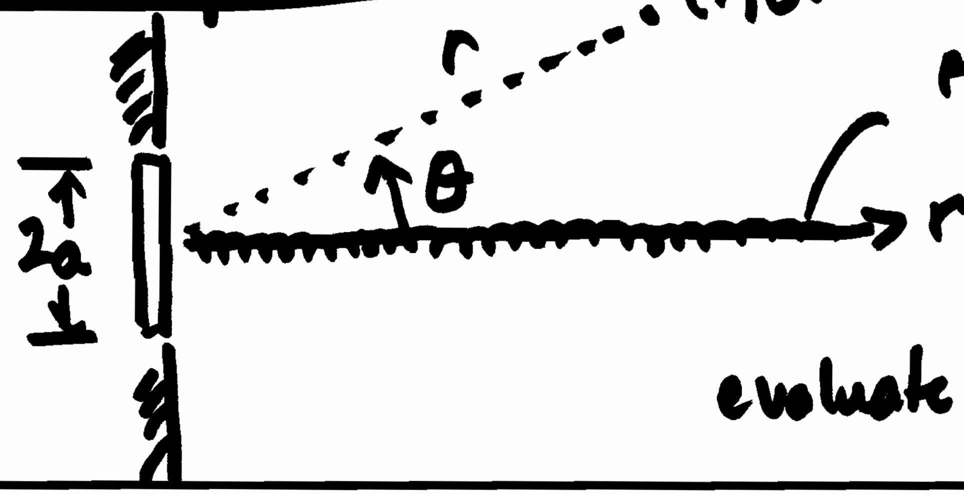
In general, it's impossible to ~~integard~~ integrate the expression for $\hat{p}(r,\theta)e^{j\omega t}$.

However, there are two exceptions, or approximations that give insight.

On-Axis Response



we restrict attention to $(r,\theta) = (r,0)$

Then, we can evaluate the integral.

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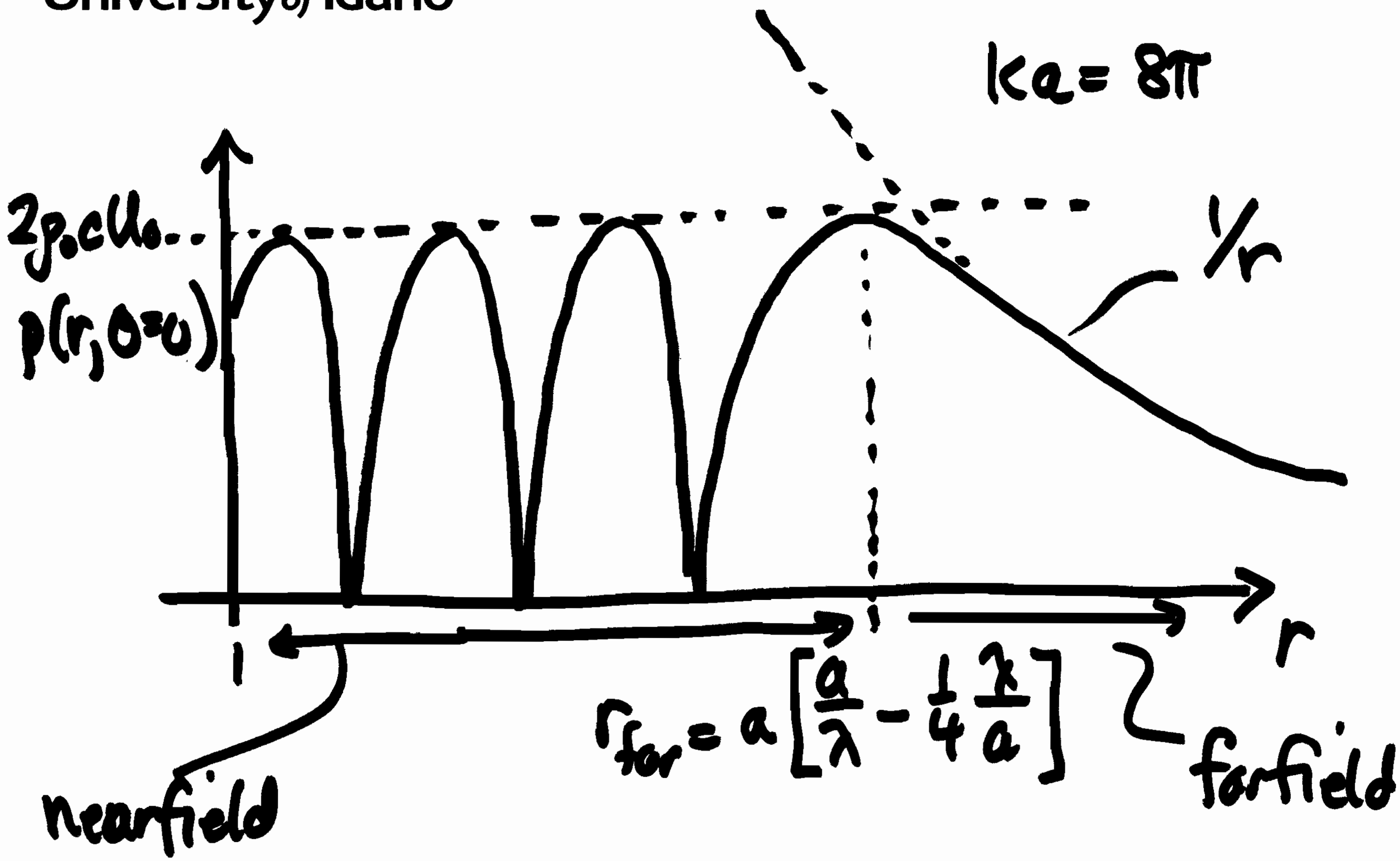
This reduces to

$$\hat{p}(r,\theta)e^{j\omega t} = \rho_0 c U_0 \left\{ 1 - \exp\left[-jk\left(\sqrt{r^2+a^2} - 1\right)\right\} e^{j(\omega t - kr)}$$

$$p(r,\theta) = |\hat{p}(r,\theta)| = 2\rho_0 c U_0 \left| \sin\left\{ \frac{1}{2}kr\left[\sqrt{1+\left(\frac{a}{r}\right)^2} - 1\right]\right\}\right|$$



$r_{far} = a\left[\frac{a}{\lambda} - \frac{1}{4}\frac{\lambda}{a}\right]$

In general, r_{far} grows with frequency.

- If $ka = \pi$; $r_{far} = 0$

- If $r >> r_{far}$, then ~~the~~ an approximation for the pressure amplitude is

$$\cancel{\hat{p}(r,\theta) =}\ \hat{p}(r,\theta)e^{j\omega t} \approx j\frac{\rho_0 c}{2} U_0 \frac{a}{r} ka\, e^{j(\omega t - kr)}$$

$1/r$

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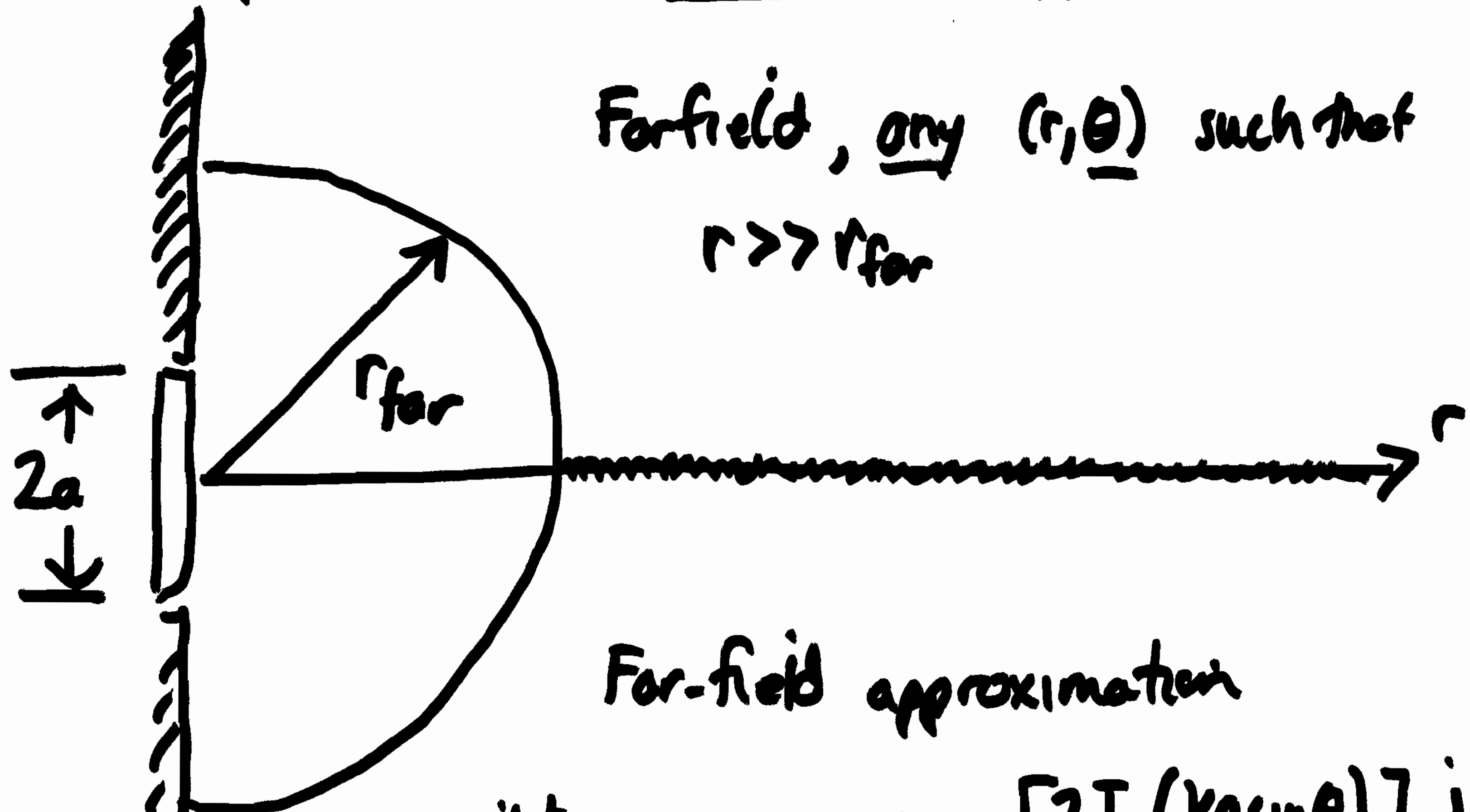
A second approximation ⇒ <u>Far-Field Approximation</u>



Farfield, <u>any</u> $(r, \underline{\theta})$ such that

$r >> r_{far}$

Far-field approximation

$$\hat{p}(r,\theta)e^{j\omega t} \approx j\frac{\rho_0 c}{2} U_0 \frac{a}{r} ka \underbrace{\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]}_{\text{directivity factor}} e^{j(\omega t - kr)}$$

$J_1(x)$ is a Bessel function of the first kind, order 1

$$\frac{J_1(x)}{x} = \mathrm{jinc}(x) \quad ; \quad \frac{\sin(x)}{x} = \mathrm{sinc}(x)$$

What is $\dfrac{2J_1(ka\sin\theta)}{ka\sin\theta}$

For fixed ka

$$\theta = 0^\circ \Rightarrow ka\sin\theta = 0 \Rightarrow \frac{2J_1(0)}{0} = 1$$

What happens as θ increases; $ka\sin\theta$ increases

When $ka\sin\theta = 3{,}73$ $\quad \hat{p}(r,\theta)e^{j\omega t} = 0$