University of Idaho First Step is to convert the incident plane wave into spherical coordinates

$$e^{j(\omega t - kr\cos\theta)} = e^{j\omega t} \sum_{m=0}^{\infty} (2m+1)(-j)^m P_m^0(\cos\theta)\, j_m(kr)$$

$$\hat{p}_c e^{j\omega t} = e^{j\omega t} \sum_{m=0}^{\infty} (2m+1)(-j)^m P_m(\cos\theta)\, j_m(kr) \quad \&$$

$$\hat{p}_s e^{j\omega t} = \sum_{n=0}^{\infty} A_n h_n^{(2)}(kr) P_n(\cos\theta)\, e^{j\omega t}$$

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$$\cancel{\rho_0 j\omega}\, \hat{u}_{rL} = (2\cdot 0+1)(-j)^0 \overbrace{P_0(\cos\theta)}^{1}\left[-k\, j_1(kr)\right]$$

$$+ \sum_{n=1}^{\infty} (2n+1)(-j)^n P_n(\cos\theta) \frac{k}{2n+1}\left[n j_{n-1}(kr) - (n+1) j_{n+1}(kr)\right]$$

$$\hat{u}_{rL} = \frac{k j_1(kr)}{\rho_0 j\omega} - \frac{1}{\rho_0 j\omega}\left[\,\cdot\,\right]$$

$$\hat{u}_{rs} = -\frac{1}{\rho_0 j\omega}\left[-A_0 k h_1^{(2)}(kr) + \sum_{n=1}^{\infty} A_n P_n(\cos\theta) \frac{k}{2n+1}\left[\,\cdot\,\right]\right]$$

$$n h_{n-1}^{(2)}(kr) - (n+1) h_{n+1}^{(2)}(kr)$$

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$$\hat{u}_{rs} = \frac{A_0 k\, h_1^{(2)}(kr)}{\rho_0 j\omega} - \frac{1}{j}\sum_{n=1}^{\infty} \frac{A_n k}{2n+1} P_n(\cos\theta)\left[n h_{n-1}^{(2)}(kr) - (n+1) h_{n+1}^{(2)}(kr)\right]$$

$\rho_0 j\omega$

$$\boxed{\begin{aligned} A_0 &= -\frac{\rho_0 K j_1(ka)}{K h_1^{(2)}(ka)} = -\frac{j_1(ka)}{h_1^{(2)}(ka)} \qquad n=0 \\ A_n &= -\frac{(2n+1)(-j)^n\left[n j_{n-1}(ka) - (n+1) j_{n+1}(ka)\right]}{n h_{n-1}^{(2)}(ka) - (n+1) h_{n+1}^{(2)}(ka)} \qquad n>0 \end{aligned}}$$

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$$\left.\begin{aligned} \hat{p}_c &= \sum_{n=0}^{\infty} (2n+1)(-j)^n P_n(\cos\theta)\, j_n(kr) \\ \hat{p}_s &= \sum_{n=0}^{\infty} A_n P_n(\cos\theta)\, h_n^{(2)}(kr) \end{aligned}\right\} \hat{p} = \hat{p}_c + \hat{p}_s$$