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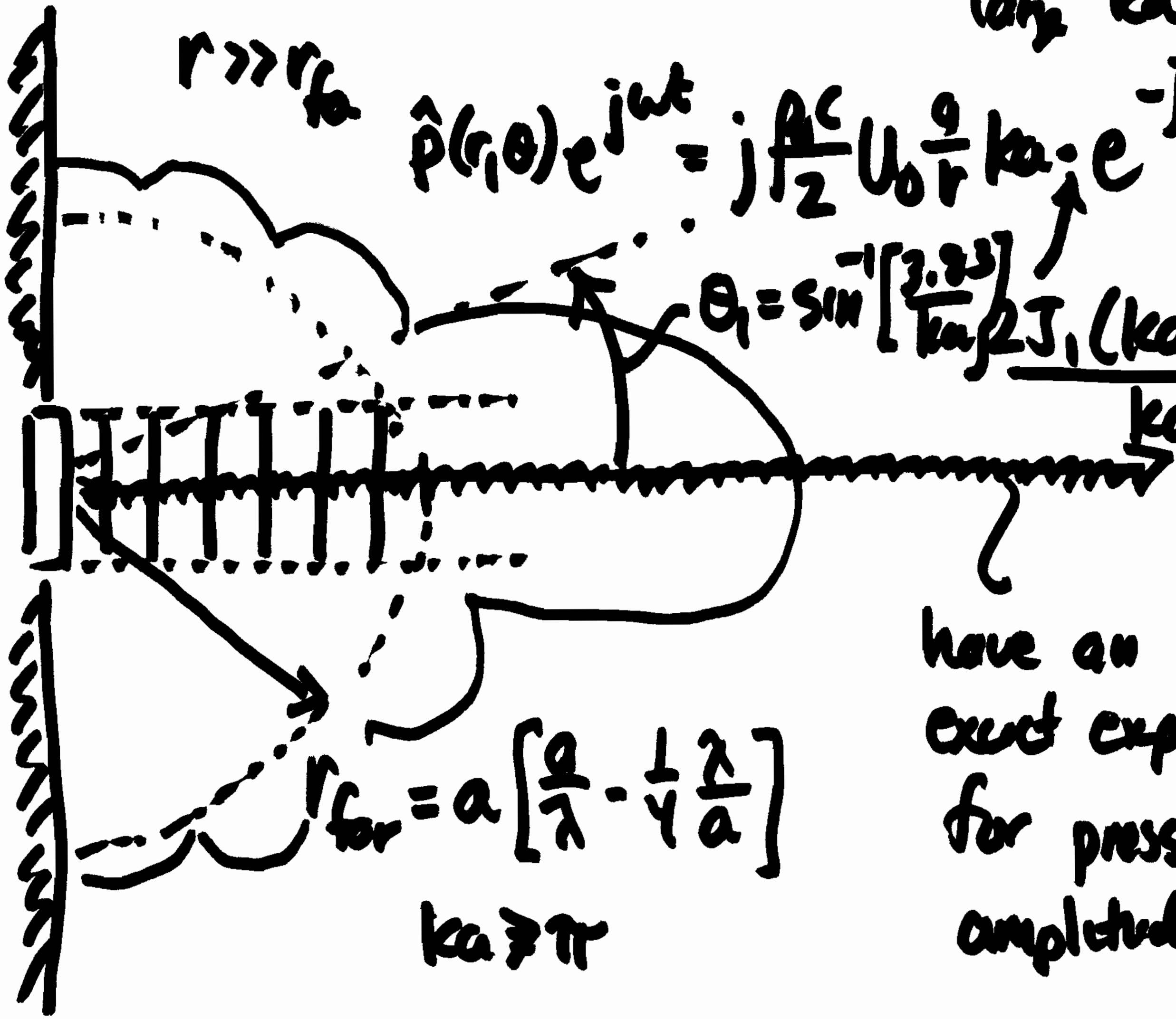
Summary of Plane Piston Radiator

For high ka, near field reduces to collimated tube of plane waves



$r \gg r_{far}$

"large ka"

$$\hat{p}(r,\theta)e^{j\omega t} = j\frac{\rho_0 c}{2}U_0\frac{a}{r}ka \cdot \frac{2J_1(ka\sin\theta)}{ka\sin\theta}e^{-jkr}e^{j\omega t}$$

$\theta_1 = \sin^{-1}\left[\frac{3.83}{ka}\right]$

have an exact expression for pressure amplitude

$$r_{far} = a\left[\frac{a}{\lambda} - \frac{1}{4}\frac{\lambda}{a}\right]$$

$ka \geq \pi$

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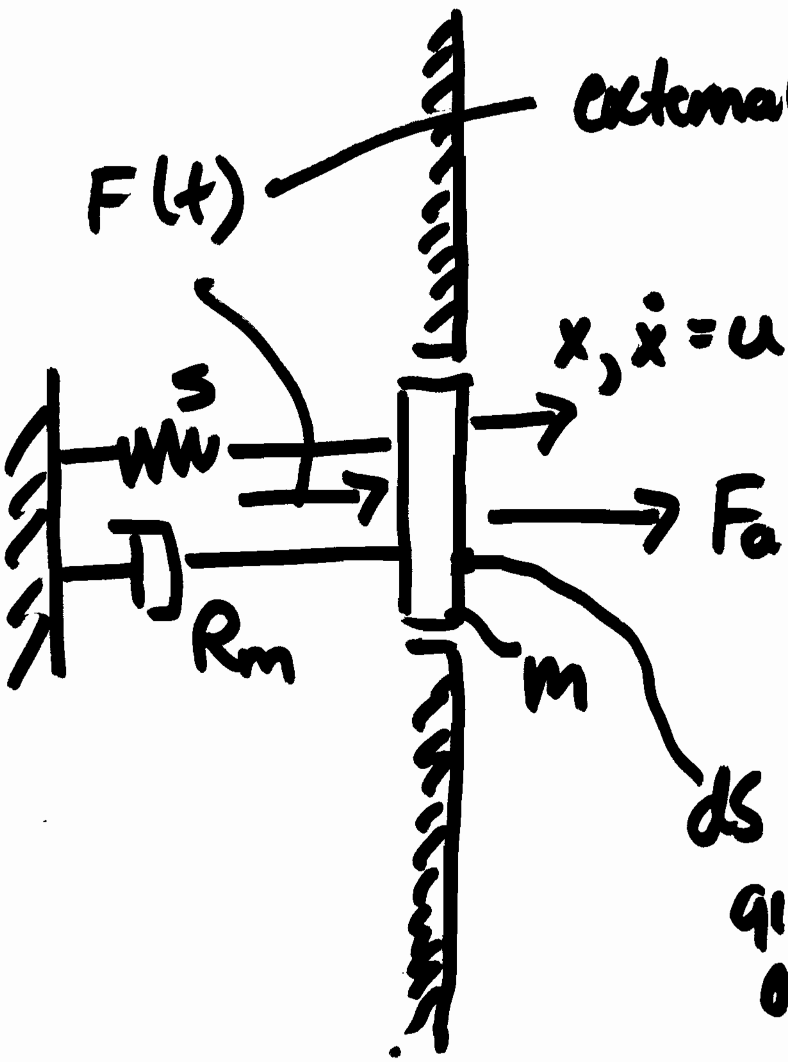
Radiation Impedance for a ~~Cir~~ Circular Plane Piston



externally applied force

$F_a(t) =$ force of air on piston $= -\int_S p(S,t)\,dS$

dS is a given differential area on the piston

The equation of motion for the piston is;

$$F(t) - sx - R_m\dot{x} + F_a(t) = m\ddot{x}$$

As before, change dependent variable from x to u;

$u = \int x\,dt$;

$$F(t) - s\int u\,dt - R_m u + F_a(t) = m\dot{u}$$

Now, restrict ourselves to the case that $F(t)$ is harmonic

$F(t) = F\cos\omega t \Rightarrow u(t)$ is also harmonic

Make replacements

$u(t) \to \hat{u} e^{j\omega t}$, $F(t) \to \hat{F} e^{j\omega t}$, $\hat{F} = F + 0j$

$f_a(t) \to \hat{F}_a e^{j\omega t}$

Gives

$$\hat{F} + \hat{F}_a = \left[(j\omega)m + R_m + \frac{s}{j\omega}\right]\hat{u}$$

Given that $f_a(t)$ is harmonic, it turns out the radiation reactance the integral for $F_a(t)$ reduces to

$$F_a(t) = -\int_S p_a(S,t)\,dt \Rightarrow \hat{F}_a = -\hat{Z}_r \hat{u} = -(R_r + jX_r)\hat{u}$$

radiation impedance radiation resistance

Put $\hat{F}_a$ into the equation of motion;

$$\hat{F} - R_r\hat{u} - jX_r\hat{u} = (j\omega)m\hat{u} + R_m\hat{u} + \frac{s}{j\omega}\hat{u}$$

$$\hat{F} = \left[j\omega\frac{X_r}{\omega} + j\omega m\right]\hat{u} + \left[R_m + R_r\right]\hat{u} + \frac{s}{j\omega}\hat{u}$$

$m_r = \frac{X_r}{\omega}$ = radiation mass

radiation damping

Finally:

$$\hat{F} = (j\omega)(m+m_r)\hat{u} + (R_m+R_r)\hat{u} + \frac{s}{j\omega}\hat{u}$$

How do we calculate m_r and R_r ??

$$R_r = \underbrace{\pi a^2}_{\text{area of x-ducer}} \rho_0 c \, R_1(2ka) \quad ; \quad R_1(2ka) = 1 - \frac{2J_1(2ka)}{2ka}$$

$$\cancel{X_r = \frac{4}{\pi}\left(\frac{x}{3} - \frac{x^3}{3^2 \cdot 5} + \frac{x^5}{3^2 \cdot 5^2 \cdot 7} + \dots\right)}$$

$$X_r = \pi a^2 \rho_0 c \, X_1(2ka)$$

$$X_1 = \frac{4}{\pi}\left(\frac{x}{3} - \frac{x^3}{3^2 \cdot 5} + \frac{x^5}{3^2 \cdot 5^2 \cdot 7} + \dots\right)$$

Problem context:

Given; $a, \rho_0, c, m, R_m, s, f, F$ ← properties of medium, piston forcing.

$\Rightarrow$ Compute $2ka$

$\Rightarrow$ Calculate R_r, m_r

$\Rightarrow$ From $\hat{F} = j\omega(m+m_r)\hat{u} + (R_m+R_r)\hat{u} + \frac{s}{j\omega}\hat{u}$
calculate $\hat{u}$.

$\Rightarrow$ $\hat{u} \Leftrightarrow U_0$ use plane piston near or far field expressions to compute acoustic pressure.

$$\hat{p}(r,\theta)e^{j\omega t} = j\frac{\rho_0 c}{2}\hat{u}\frac{a}{r}ka\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]e^{-jkr}e^{j\omega t}$$