Sound Power Output of a Circular Plane Piston Radiator....

w/o proof

$$\Pi = \text{Time avg'd acoustic power output} = \frac{1}{2} R_r U_0^2 \text{ in Watts}$$

U_0 is the vibration velocity amplitude of the vibrating piston.

Low and High-Frequency Asymptotes for Radiation Impedance....

Low frequency limit $2ka \ll 1$ (small piston)

$$R_r \approx \frac{1}{2}\rho c\, \underset{\substack{\uparrow \\ \text{area of} \\ \text{transducer}}}{\pi a^2}\, (ka)^2$$ since $ka \ll 1$, weak radiator

$$X_r \approx \frac{8}{\pi}\rho c\, \pi a^2 (ka) \Rightarrow m_r = \frac{X_r}{\omega} \approx \rho_0\, \pi a^2 \frac{8a}{3\pi} = \rho\, \underline{\underline{\pi a^2}}\, \underline{\underline{(0.85a)}}$$

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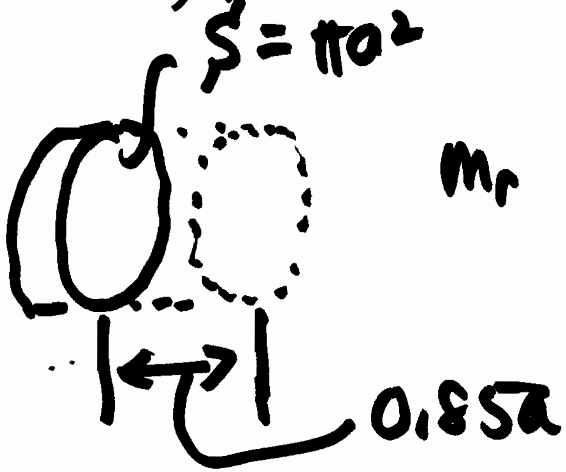

m_r is same as mass of medium in this volume.

In the high-frequency limit; $2ka >> 1$

$R_r \approx \rho_0 c \pi a^2$ $\qquad R_1(2ka) \approx 1$

$X_r \approx 0$

Well, $\Pi = \begin{array}{c}\text{Time avg}\\\text{power}\\\text{output}\end{array} = \frac{1}{2} R_r |\hat{u}|^2 \Rightarrow \Pi \approx \frac{1}{2} \rho_0 c \pi a^2 |\hat{u}|^2$

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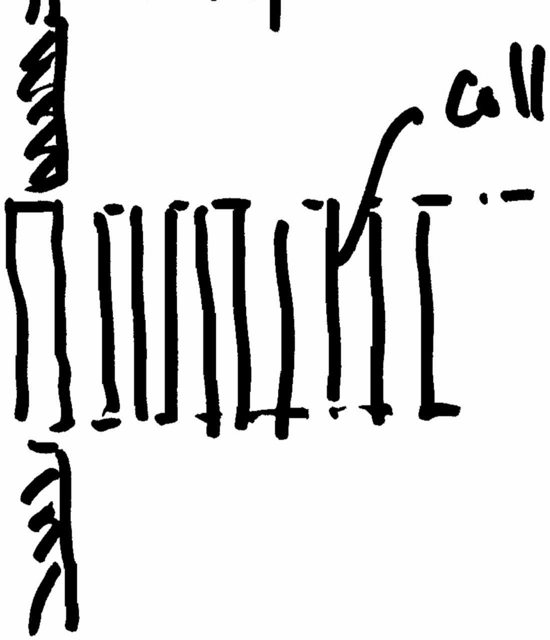
Physically, at high ka:



collimated tube of plane waves in nearfield.

$|\hat{p}|$ acoustic pressure amplitude

For plane waves $I = \dfrac{|\hat{p}|^2}{2\rho_0 c}$ $\quad \hat{u} = \dfrac{\hat{p}}{\rho_0 c}$ $\quad |\hat{p}| = \rho_0 c\,|\hat{u}|$

$$= \frac{(\rho_0 c)^2 |\hat{u}|^2}{2\rho_0 c} = \frac{1}{2}\rho_0 c\,|\hat{u}|^2$$

$$\Pi = I\,\pi a^2 = \frac{1}{2}\rho_0 c\,\pi a^2 |\hat{u}|^2$$

assume plane waves only in front of transducer