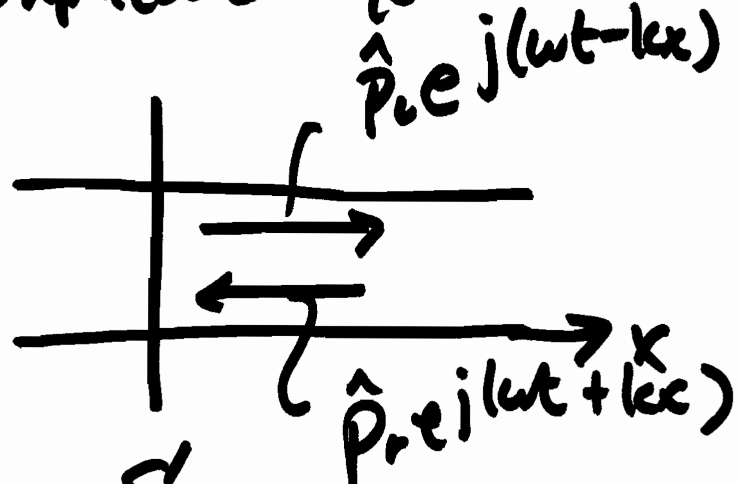
Back to Propagation in Pipes

Adopt problem-solving ~~staged~~ strategy used for complicated systems...



S ≈ cross-sectional area.

~~the~~ Invoke the narrow pipe assumption

$$\lambda \gg \sqrt{\frac{S}{\pi}}$$

$$\hat{p}(x,t) = \hat{p}_i e^{j(\omega t - kx)} + \hat{p}_r e^{j(\omega t + kx)} \quad ; \text{Pa}$$

$$\hat{u}(x,t) = \frac{\hat{p}_i}{\rho_0 c} e^{j(\omega t - kx)} - \frac{\hat{p}_r}{\rho_0 c} e^{j(\omega t + kx)} \quad ; \text{m/s}$$

In working pipe problems, its customary to use volume velocity instead of acoustic velocity;

$$\text{Volume Velocity} = \hat{U}(x,t) = \hat{u}(x,t)\,S \qquad \text{m}^3/\text{s}$$

Use acoustic pressure as before, only now $\hat{U}(x,t)$ is

$$\hat{U}(x,t) = \frac{\hat{p}_i}{\rho_0 c/S} e^{j(\omega t - kx)} - \frac{\hat{p}_r}{\rho_0 c/S} e^{j(\omega t + kx)}$$

~~= Acoustic~~

Furthermore; we define <u>acoustic impedance</u> $\hat{Z}$ as

$$\hat{Z} = \frac{\hat{p}}{\hat{U}} = \frac{\text{acoustic pressure}}{\text{acoustic volume velocity}} = \text{acoustic impedance}$$

- $\hat{z} = \dfrac{\hat{p}}{\hat{u}}$ = specific acoustic impedance.

lower case

<u>For a rightward travelling</u> wave (by itself) (also called progressive wave)

$$\hat{z} = \rho_0 c = z = r = \text{characteristic impedance of the medium.}$$

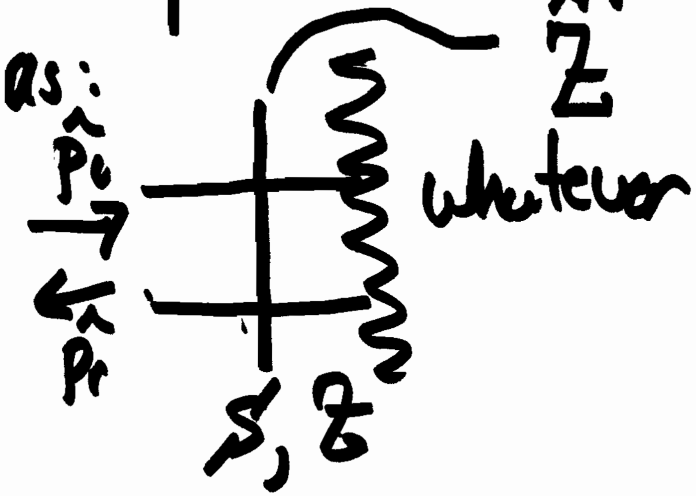
and

$$\hat{Z} = \frac{\hat{p}}{\hat{U}} = \frac{\hat{p}}{\underbrace{\hat{u}}_{\text{acoustic velocity for RTPW}} S} = \frac{\cancel{\hat{p}}}{\frac{\cancel{\hat{p}}}{\rho_0 c} \cancel{S}} = \frac{\rho_0 c}{\cancel{S}} = \text{characteristic acoustic impedance of the pipe}$$

$$= Z$$

In general we approach the modeling and design problem

as:



$$\frac{\hat{p}_i + \hat{p}_r}{\hat{U}_i + \hat{U}_r} = \hat{Z} = \frac{\hat{p}_i + \hat{p}_r}{\frac{\hat{p}_i}{Z} - \frac{\hat{p}_r}{Z}}$$

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Multiply right-hand side by $\frac{1/\hat{p}_c}{1/\hat{p}_c}$ $\qquad \hat{R} = \hat{p}_r / \hat{p}_c$

$$\hat{Z} = \frac{1+\hat{R}}{\frac{1}{Z} - \frac{\hat{R}}{Z}} \quad \text{Solve for } \hat{R} \Rightarrow \hat{R} = \frac{\hat{Z} - Z}{\hat{Z} + Z}$$

$$Z = \frac{\rho c}{S}$$

R_I = Intensity reflection coefficient $= \frac{I_r}{I_i} = |\hat{R}|^2 = \frac{I_r S}{I_i S} = \frac{\Pi_r}{\Pi_i} = R_\Pi$

$$T_\Pi = \frac{\Pi_t}{\Pi_i} = 1 - |\hat{R}|^2$$

Design metrics

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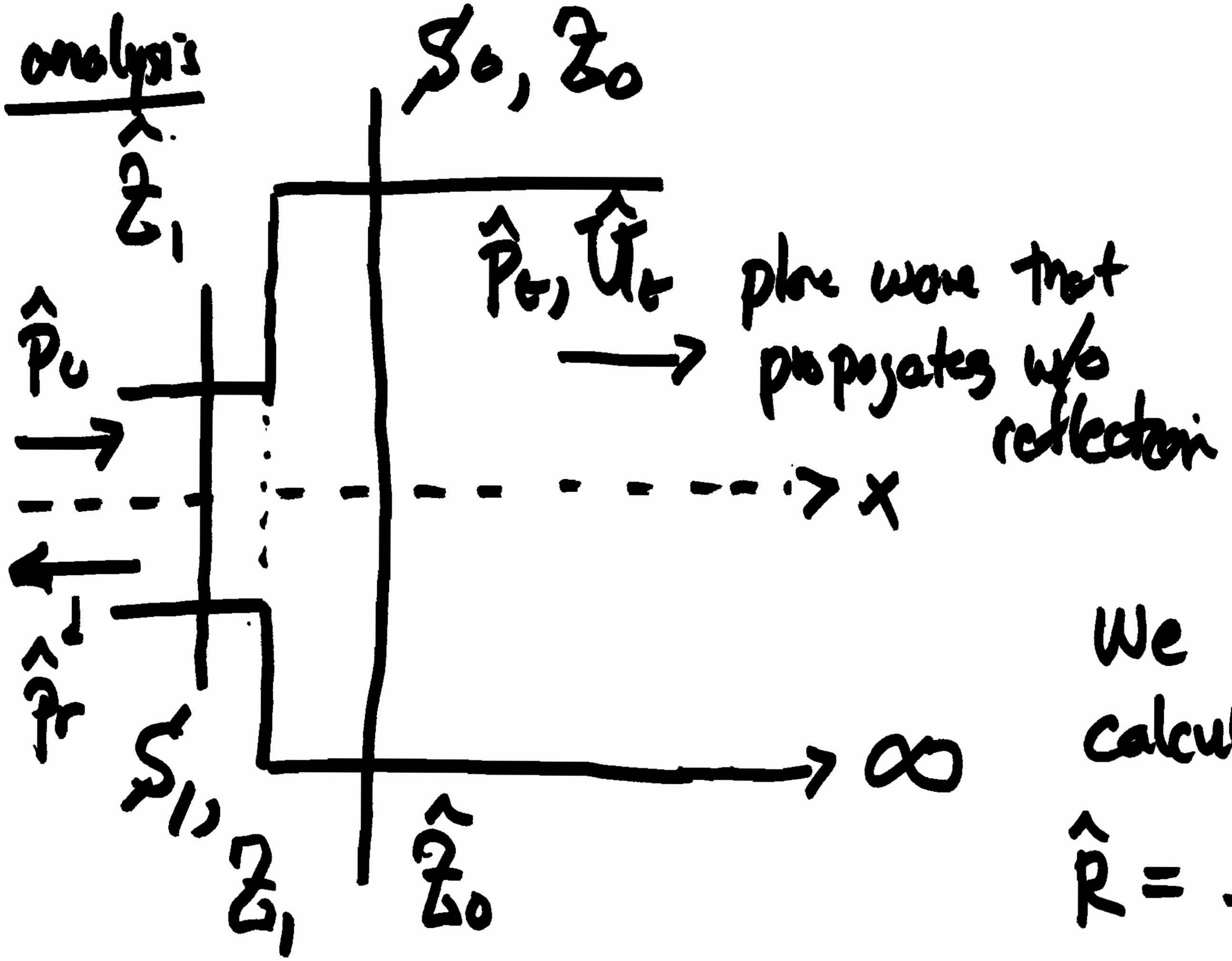
Lets consider one acoustic example and build tools for analysis



We wish to calculate

$$\hat{R} = \frac{\hat{Z}_1 - Z_1}{\hat{Z}_1 + Z_1} \; ; \; Z_1 = \frac{\rho_0 c}{S_1}$$

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We also know that $\hat{Z}_D = \hat{Z}_0$ because RTPW in downstream section.

For the area change at $x=0$, we determine $\hat{Z}_1$ by applying the following physical boundary conditions;

divide the two

continuity in acoustic pressure $\Rightarrow \hat{p}_i + \hat{p}_r = \hat{p}_t$

$$\frac{\hat{p}_i + \hat{p}_r}{\hat{U}_i + \hat{U}_r} = \frac{\hat{p}_t}{\hat{U}_t}$$

continuity in acoustic volume velocity $\Rightarrow \hat{U}_i + \hat{U}_r = \hat{U}_t$

$$\Downarrow$$

$\hat{Z}_1 = \hat{Z}_0 \leftarrow$ continuity in acoustic impedance across an area change

Now we can solve the problem:

① We start with the "termination impedance"

$$\hat{Z}_0 = \begin{array}{c}\text{termination}\\ \text{impedance}\end{array} = Z_0 \quad \text{because termination is RTPW}$$

② Invoke continuity in acoustic impedance across the area change $0 \to 1$

$$\hat{Z}_1 = \hat{Z}_0 = Z_0 \qquad Z_0 = \frac{\rho c}{S_0} \; ; \; Z_1 = \frac{\rho c}{S_1}$$

$$\therefore \quad \hat{R} = \frac{Z_0 - Z_1}{Z_0 + Z_1} = \frac{S_1 - S_0}{S_1 + S_0}$$