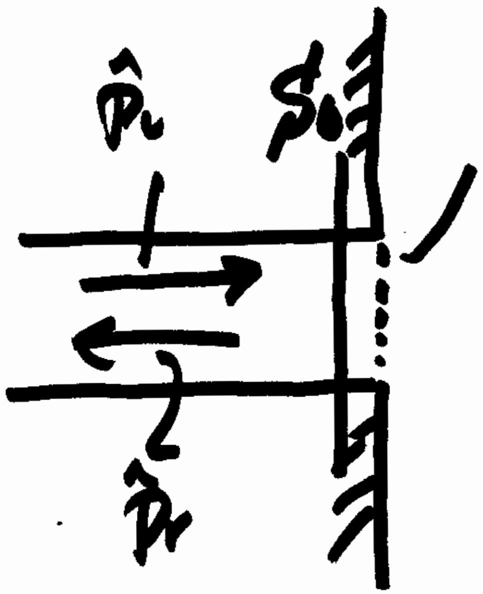
Radiation from an open pipe



imagine a massless piston

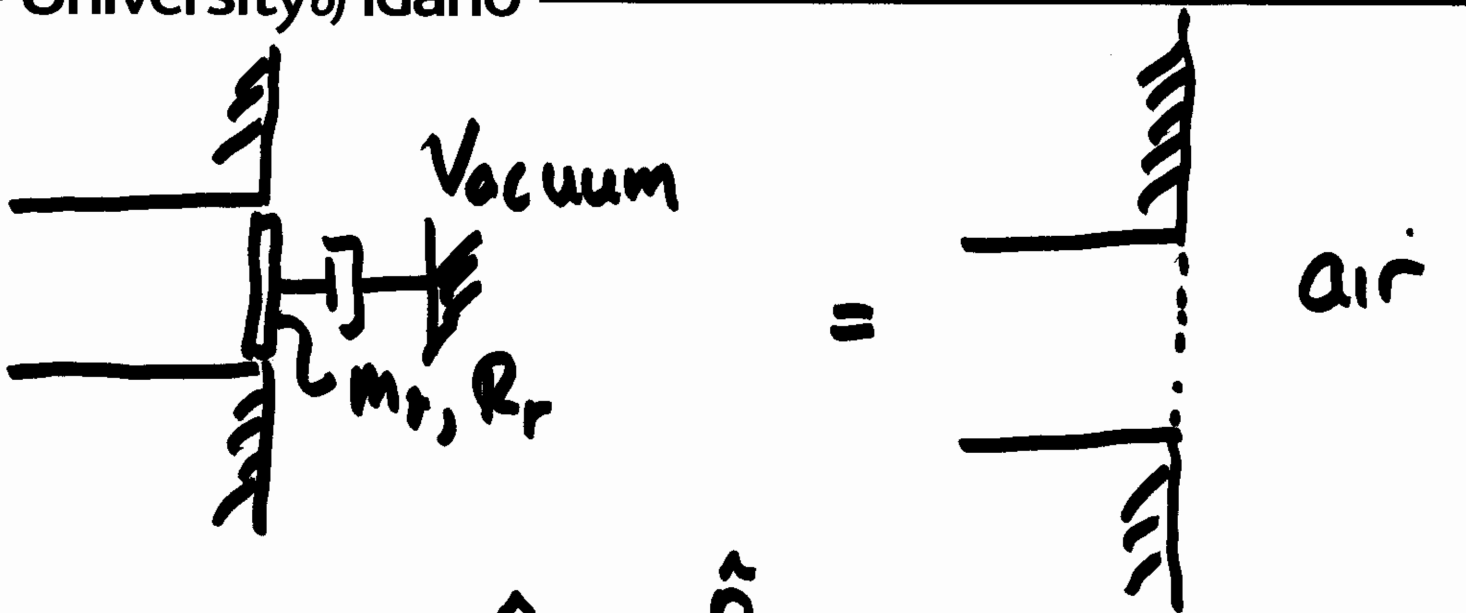
Remember, for a rigid piston, and harmonic motion

$\hat{F}_a = -\hat{Z}_r \hat{u}$ $\quad$ $\hat{Z}_r$ = radiation impedance for rigid (baffled) piston

Force on fluid $\hat{F} = -\hat{F}_a$, so

$$\hat{Z}_r = \frac{\hat{F}}{\hat{U}} = \frac{\hat{p} S_0}{\hat{u}} = \frac{\hat{p} S_0}{\hat{U}/S_0} = \frac{\hat{p}}{\hat{U}} \cdot S_0^2 = \hat{Z}_0 S_0^2$$

$\hat{Z}_0$ → acoustic impedance at S_0



So; we get $\hat{Z}_0 = \frac{\hat{Z}_r}{S_0^2}$

Also, for a baffled plane piston,

$\hat{Z}_r = j\omega m_r + R_r \Rightarrow \hat{Z}_0 = j\omega \frac{m_r}{S_0^2} + \frac{R_r}{S_0^2}$

↗ valid for any frequency

But, we want to have $\lambda >> D$; $ka << 1$ for the opening ;

R_r $\quad jX_r = j\omega \frac{X_r}{\omega} = j\omega m_r$

$$Z_r = \begin{cases} \frac{1}{2}\rho_0 c S (ka)^2 + j\frac{8}{3\pi}\rho_0 c S (ka) & \text{flanged (baffle)} \\ \frac{1}{4}\rho_0 c S (ka)^2 + j\rho_0 c S (0.6)(ka) & \text{unflanged} \end{cases}$$

Levine and Schwinger (1946?)

In terms of a acoustic impedance $(ka \ll 1)$

$$\hat{z}_0 = \frac{\hat{Z}_r}{S_0^2} = \begin{cases} \frac{1}{2}\rho_0 c \frac{(ka)^2}{S_0} + j\frac{8}{3\pi}\frac{\rho_0 c}{S_0}(ka) & \text{flanged} \\ \frac{1}{4}\frac{\rho_0 c}{S_0}(ka)^2 + j0.6\frac{\rho_0 c}{S_0}(ka) & \text{unflanged.} \end{cases}$$

$\Rightarrow$ Termination Impedance of Open Pipe; Flanged or Unflanged.