Wideband Sound Measurement

Say we measure a wide-band "noise". We start with a batch of sampled microphone voltages;

V_n ; $n=0,1,\ldots,N-1$; $\bar{V}=0$; $\Delta t =$ sampling interval

wish to compute Sound Pressure Level: $f_s = \frac{1}{\Delta t} =$ sampling frequency

Given DFT decomposition

$$V_n \rightarrow \boxed{DFT} \rightarrow \tilde{V}_k$$

Then, we visualize the microphone voltage as

$$\tilde{V}(n\Delta t) = \sum_{k=1}^{\frac{N}{2}-1} \left|\frac{2}{N}\hat{V}_k\right| \cos(2\pi f_k t + \phi_k)$$

$$+ \frac{1}{N} \hat{V}_{\frac{N}{2}} \cos\left(2\pi f_{\frac{N}{2}} t + \phi_{\frac{N}{2}}\right)$$

$\hat{V}_{\frac{N}{2}}$ is real valued.

$f_{\frac{N}{2}} = \frac{f_s}{2}$ = Nyquist frequency

Recall, that for a pure tone

$$M_f = \frac{V}{P}$$

So, at each frequency f_k, we can compute an acoustic pressure amplitude P_k with .

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$\cancel{P_k = V(f_k)}$ $\quad P(f_k) = \dfrac{V(f_k)}{M_f(f_k)}$ ← Free-field sensitivity at frequency f_k;

We can then visualize the acoustic pressure as

$$p(n\Delta t) = \sum_{k=1}^{\frac{N}{2}-1} P_k \cos(2\pi f_k t + \phi_k + \gamma_k)$$

$$+ P_{\frac{N}{2}} \cos(2\pi f_{\frac{N}{2}} t + \phi_{\frac{N}{2}} + \gamma_{\frac{N}{2}})$$

phase shift caused by microphone

$$P_k = \frac{1}{M_f(f_k)} \left| \frac{2}{N} \hat{V}_k \right| \quad ; \quad P_{\frac{N}{2}} = \frac{1}{M_f(f_{\frac{N}{2}})} \frac{1}{N} \hat{V}_{\frac{N}{2}}$$

Then, according to Parseval's theorem;

$$P_{rms}^2 = \frac{1}{N}\sum_{n=0}^{N-1} p_n^2 = \frac{1}{2}\cdot\sum_{k=1}^{\frac{N}{2}-1} P_k^2 + \frac{1}{2}\left(\sqrt{2}\,P_{\frac{N}{2}}\right)^2$$

Then;

$$SPL = 20\lg\left[\frac{P_{rms}}{P_{ref}}\right]$$

Frequency-weighted by microphone sensitivity

What if we wish to compute A-Weighted SPL of a wide-band signal?? Lets start with

$$p(n\Delta t) = \sum$$

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Then,

$$p_a(n\Delta t) = \sum_{k=1}^{\frac{N}{2}-1} P_k \cos(2\pi f_k t + \phi_k) + P_{\frac{N}{2}} \cos(2\pi f_{\frac{N}{2}} t + \phi_{\frac{N}{2}})$$

$$P_k = \frac{W(f_k)}{M_f(f_k)} \left| \frac{2}{N} \hat{V}_k \right| \quad k = 0, 1, \ldots \frac{N}{2} - 1$$

$$P_{\frac{N}{2}} = \frac{W(f_{N/2})}{M_f(f_k)} \frac{1}{N} \hat{V}_{\frac{N}{2}} \quad ; \ k = \frac{N}{2}$$

with this definition

$$SPL = 20 \log \left[\frac{P_{rms}}{P_{ref}} \right] \quad ; \text{ unit "dBA re } P_{ref}\text{"}$$