University of Idaho To proceed, we will replace $x = \int u\,dt$ (u = velocity of diaphragm), and we'll neglect the inductance L.

$$(Bl)\,i = s\int u\,dt + R_m u + m\dot{u}$$

$$V(t) = Ri + (Bl)\,u$$

Perform a harmonic analysis, i.e, seek steady state $i(t)$, $u(t)$, given that $V(t) = V\cos\omega t$

$$V(t) \to \hat{V}e^{j\omega t},\quad i(t) \to \hat{I}e^{j\omega t},\quad u(t) \to \hat{U}e^{j\omega t}$$

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Substituting into the two differential equations.

$$Bl\hat{I} = \frac{s}{j\omega}\hat{U} + R_m\hat{U} + (j\omega)m\hat{U}$$

$$\hat{V} = R\hat{I} + Bl\hat{U}$$

We solve for $\hat{I}$ in the second, put into the first, and obtain

$$\hat{U} = \frac{Bl/R \, \hat{V}}{(j\omega)m + \left[R_m + \frac{Bl^2}{R}\right] + s/j\omega}$$

University of Idaho Define

$\omega_n = \sqrt{\frac{s}{m}}$ = "free-air natural frequency"

$f_n = \frac{\omega_n}{2\pi} = \underline{\underline{f_s}}$ (terminology used in loudspeaker industry).

$\underline{\underline{Q_m}} = \frac{s}{R_m \omega_n}$ = Mechanical Quality Factor, "Mechanical Q". (dimensionless)

$\underline{\underline{Q_e}} = \frac{R_s}{B\ell^2 \omega_n}$ = Electrical Quality Factor, "Electrical Q". (dimensionless)

University of Idaho and $\frac{1}{Q_m} + \frac{1}{Q_e} = \frac{1}{Q_t}$

where $\underline{\underline{Q_t}}$ = Total Quality Factor (dimensionless)

Then: The velocity amplitude of the loudspeaker moving mass is:

$$\hat{U} = \frac{Bl\,\omega_n}{R_s} \frac{1}{\left(j\frac{\omega}{\omega_n}\right) + \underbrace{\left(\frac{1}{Q_m} + \frac{1}{Q_e}\right)}_{= \frac{1}{Q_t}} + \left(\frac{\omega_n}{j\omega}\right)} \hat{V}$$

The underlined qty's are known as the "Thiele / Small" parameters

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Applying the same steps to the loudspeaker driver that we did with the mass-spring-damper, we will get

$$u(t) = \frac{Bl}{R} V \frac{Q_t}{m\omega_n} \frac{1}{\sqrt{1 + Q_t^2\left(\frac{\omega}{\omega_n}\right)}}$$

$$= \underbrace{\frac{Bl}{R} V \frac{Q_t}{m\omega_n} \frac{1}{\sqrt{1 + Q_t^2\left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega}\right)^2}}}_{|\hat{u}|} \cos(\omega t + \arg \hat{u})$$

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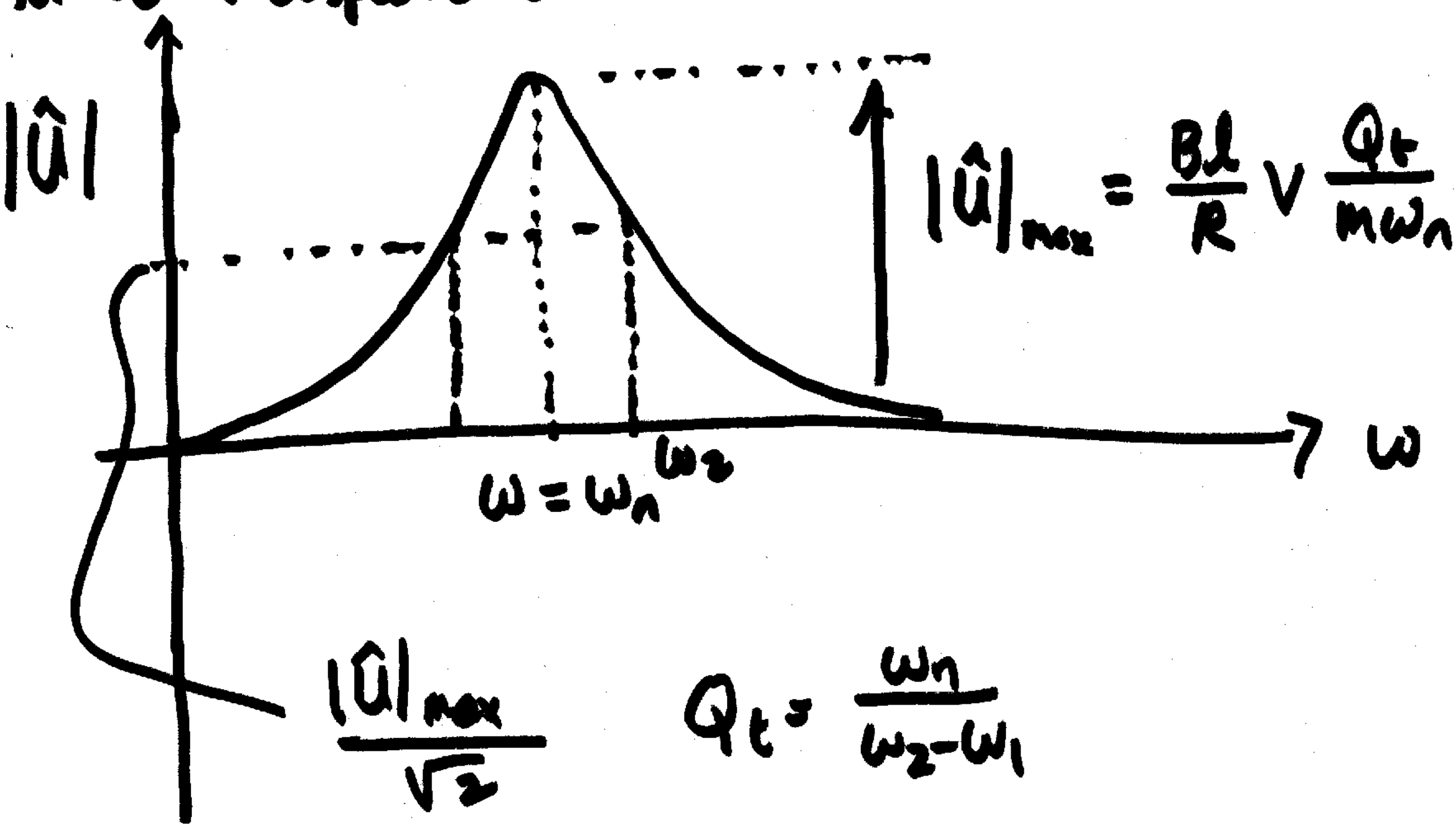
Then, the frequency response diagram for the loudspeaker driver will look like



$$Q_t = \frac{\omega_n}{\omega_2 - \omega_1}$$