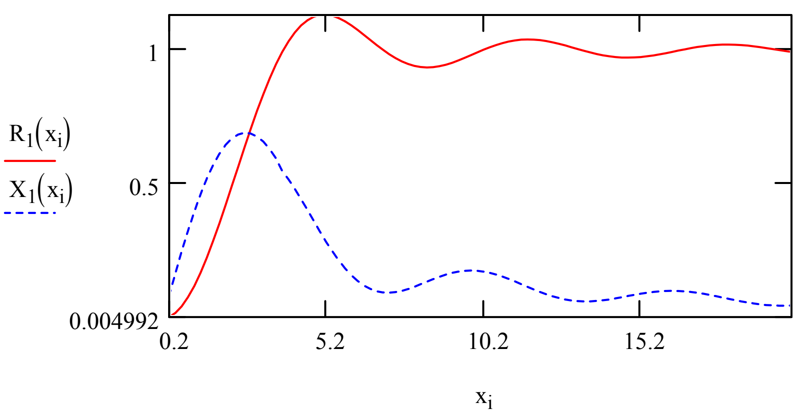
Radiation Impedance for Circular Plane Piston

Functions for Radiation Resistance, Reactance. Asymptotic forms for X1(x) can be found in Pierce.

$$R_1(x) := 1 - 2\cdot\frac{J1(x)}{x}$$

$$X_1(x) := \text{if}\left[x < 3.75, \frac{4}{\pi}\cdot\left(\frac{x}{3} - \frac{x^3}{3^2\cdot 5} + \frac{x^5}{3^2\cdot 5^2\cdot 7} - \frac{x^7}{3^2\cdot 5^2\cdot 7^2\cdot 9}\right), \frac{4}{\pi\cdot x} + \sqrt{\frac{8}{\pi}}\cdot\frac{\sin\left(x - \frac{3\cdot\pi}{4}\right)}{x^{1.5}}\right]$$

$$i := 1, 2 .. 100 \qquad x_i := \frac{i}{100}\cdot 20$$



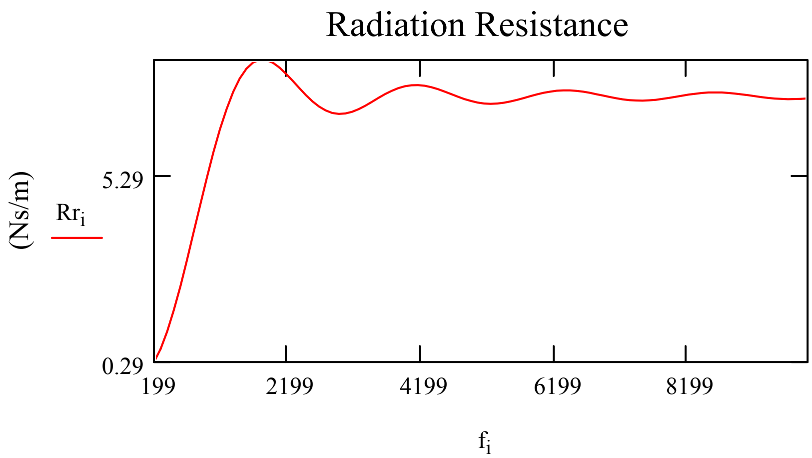
Do some numerical examples

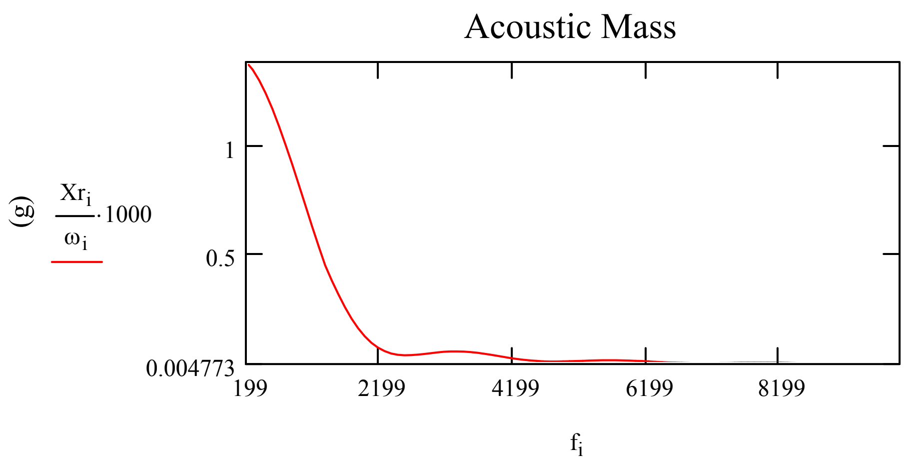
$$a := 3\cdot 0.0254 \qquad c := 340 \qquad \rho := 1.21 \qquad U_o := 0.01$$

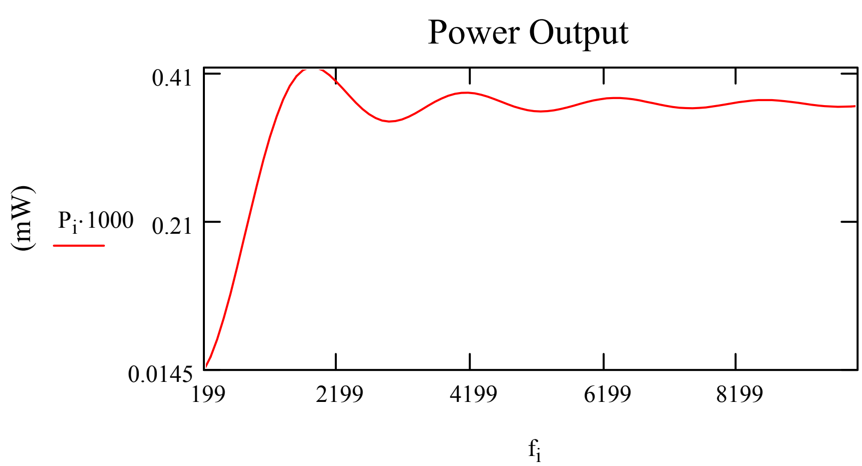
$$f_{min} := 100 \qquad f_{max} := 10000 \qquad f_i := \frac{i}{100}\cdot\left(f_{max} - f_{min}\right) + f_{min} \qquad \omega_i := 2\cdot\pi\cdot f_i$$

$$k_i := \frac{\omega_i}{c}$$

$$Xr_i := \pi\cdot a^2\cdot\rho\cdot c\cdot X_1\left(2\cdot k_i\cdot a\right) \qquad Rr_i := \pi\cdot a^2\cdot\rho\cdot c\cdot R_1\left(2\cdot k_i\cdot a\right) \qquad P_i := \frac{1}{2}\cdot U_o^2\cdot Rr_i$$


Radiation Resistance
(Ns/m)
Rr_i
5.29
0.29
199
2199
4199
6199
8199
f_i


Acoustic Mass
(g)
Xr_i / ω_i · 1000
1
0.5
0.004773
199
2199
4199
6199
8199
f_i


Power Output
(mW)
P_i · 1000
0.41
0.21
0.0145
199
2199
4199
6199
8199
f_i

Do a numerical Example Computation

$m := 5\cdot 10^{-3}$ $s := 2000$ $R_m := 3$ $a := 3\cdot .0254$ $j := \sqrt{-1}$ $\rho := 1.21$ $c := 343$

$f := 3000$ $\omega := 2\cdot\pi\cdot f$ $k := \frac{\omega}{c}$ $F := 0.1$ $r := 3$ $\theta := 30$ $\theta := \theta\cdot\frac{\pi}{180}$

$k = 54.955$ $k\cdot a = 4.188$ $2\cdot k\cdot a = 8.375$

$X_1(2\cdot k\cdot a) = 0.135$ $X_r := \pi\cdot a^2\cdot\rho\cdot c\cdot X_1(2\cdot k\cdot a)$ $X_r = 1.021$ $m_r := \frac{X_r}{\omega}$ $m_r\cdot 1000 = 0.054$

$R_1(2\cdot k\cdot a) = 0.936$ $R_r := \pi\cdot a^2\cdot\rho\cdot c\cdot R_1(2\cdot k\cdot a)$ $R_r = 7.083$

$Z_m := j\cdot\omega\cdot m + R_m + \frac{s}{j\cdot\omega}$ $Z_m = 3 + 94.142i$

$Z_r := j\cdot\omega\cdot m_r + R_r$ $Z_r = 7.083 + 1.021i$

$u := \frac{F}{Z_m + Z_r}$ $U_o := |u|$ $U_o = 1.045\times 10^{-3}$

$f_n := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{s}{m + m_r}}$ $f_n = 100.118$

$H := \frac{2\cdot J1(k\cdot a\cdot\sin(\theta))}{k\cdot a\cdot\sin(\theta)}$ $H = 0.543$ $\theta_1 := \operatorname{asin}\left(\frac{3.83}{k\cdot a}\right)$ $2\cdot\theta_1\cdot\frac{180}{\pi} = 132.301$

$P := \frac{\rho\cdot c}{2}\cdot U_o\cdot\frac{a}{r}\cdot k\cdot a$ $P = 0.023$ $SPL := 20\log\left(\frac{P}{\sqrt{2}\cdot 20\cdot 10^{-6}}\right)$ $SPL = 58.228$

$P := \frac{\rho\cdot c}{2}\cdot U_o\cdot\frac{a}{r}\cdot k\cdot a\cdot H$ $P = 0.013$ $SPL := 20\log\left(\frac{P}{\sqrt{2}\cdot 20\cdot 10^{-6}}\right)$ $SPL = 52.931$

$\Pi := \frac{1}{2}\cdot U_o^{2}\cdot R_r$ $\Pi\cdot 1000 = 3.867\times 10^{-3}$