In general, we can apply the concept to a linear system model comprised of one input, and several dependent variables.

1. Choose one dependent variable as the output. It can also be any linear combination of the dependent variables.

2. Then apply the LT to the differential equation.

3. Solve for the LT of the selected output.

4. Determine the inverse LT.
Example: 2 DOF mass-spring-damper.

\[ m_1 \ddot{x}_1 + c_2 \dot{x}_1 - c_2 \dot{x}_2 + k_2 x_1 - k_2 x_2 + c_1 x_1 + k_1 x_1 = u(t) \]

\[ m_2 \ddot{x}_2 + c_2 \dot{x}_2 - c_1 \dot{x}_1 + k_2 x_2 - k_2 x_1 = 0 \]

\( u(t) \) = a force applied to mass \( m_1 \).

Select \( x_1(t) \) as the output.

First apply LT to the system model.

\[ m_1 s^2 \tilde{x}_1(s) + c_2 s \tilde{x}_1(s) - c_2 s \tilde{x}_2(s) + k_2 \tilde{x}_1(s) - k_2 \tilde{x}_2(s) + c_1 s \tilde{x}_1(s) \]
\[ + k_1 \tilde{x}_1(s) = \tilde{u}(s) \]

\[ m_2 s^2 \tilde{x}_2(s) + c_2 s \tilde{x}_2(s) - c_1 s \tilde{x}_1(s) + k_2 \tilde{x}_2(s) - k_2 \tilde{x}_1(s) = 0 \]
From the second equation, solve for $X_2(s)$. Then substitute $X_2(s)$ into the 1st equation, and isolate $X_1(s)$ on the left-hand side. The result is

$$X_1(s) = \frac{m_2 s^2 + c_2 s + k_2}{m_1 m_2 s^4 + (c_1m_1 + c_2m_2 + c_3) s^3 + (k_1 m_2 + k_2 m_1 + k_2 m_2 + c_2) s^2 + (c_1 k_2 + c_2 k_1) s + k_1 k_2} \frac{U(s)}{T(s)}$$

In general, the transfer function for an arbitrary linear system will take the form:

$$T(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

$m \leq n$ for causality

$a_i$, $b_i$'s system parameters (mech. $g_i$, $m_i$, $c_i$, $k_i$'s etc)
Suppose we start with

\[ T(s) = \frac{b_0 s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_0} \]

By inspection, it is apparent that this transfer function will result from

\[ \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_0} \]

The following differential equation

\[ x^{(n)} + a_{n-1} x^{(n-1)} + \cdots + a_1 x + a_0 x = b_0 u + b_n u \]

\[ \text{Take LT of diff eq and neglect IC's} \]

\[ s \hat{x}(s) + a_{n-1} \hat{x}^{(n-1)}(s) + \cdots + a_1 \hat{x}(s) + a_0 \hat{x}(s) = b_0 U(s) + b_n U(s) \]

We can then derive state space representation using conventional means
State Space to Transfer Function.

Start with

\[ \dot{x}_3 = [A]x_3 + [B]\Delta u \]
\[ y = [c_3]x_3 + du \]

Apply LT and neglect initial conditions:

\[ \mathcal{L}\{\dot{x}_3\} = \mathcal{L}\{[A][x_3]\} = \mathcal{L}\{[B]\Delta u\} = s\mathcal{L}\{x_3\} = s\mathcal{L}\{x_3(0)\} = s\mathcal{L}\{x_3(0)\} = s[I]\mathcal{L}\{x_3(0)\} \]

\[ \mathcal{L}\{[A]x_3 + [B]\Delta u\} = [A]\mathcal{L}\{x_3\} + [B]\mathcal{L}\{\Delta u\} \]

\[ \mathcal{L}\{y\} = \mathcal{L}\{[c_3]x_3 + du\} \Rightarrow Y(s) = [c_3]\mathcal{L}\{x_3\} + dU(s) \]
So,
\[ s[I]X(s) = [A]X(s) + SB_3U(s) \]

Solve for \( X(s) \)
\[ s[I]X(s) - [A]X(s) = SB_3U(s) \]
\[ [s[I] - [A]]X(s) = SB_3U(s) \]

\[ X(s) = [s[I] - [A]]^{-1} SB_3U(s) \]

Put \( X(s) \) into the output equation
\[ Y(s) = SC_3 [s[I] - [A]]^{-1} SB_3U(s) + dU(s) \]

\[ Y(s) = \underbrace{[SC_3^T [s[I] - [A]]^{-1} SB_3 + d]U(s)}_{T(s)} \]